Essais sur la stabilité des accords d'assurance mutuelle

Renaud Bourlès

UNIVERSITÉ DE LA MÉDITERRANÉE (AIX-MARSEILLE II)
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Renaud BOURLÈS

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ESSAIS SUR LA STABILITÉ DES ACCORDS D'ASSURANCE MUTUELLE

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M. Hubert STAHN	Professeur, Université de la Méditerranée

Résumé

Cette thèse a pour thème central l'étude du fonctionnement des accords d'assurance mutuelle et plus particulièrement de leur stabilité dans trois contextes différents.

Le premier chapitre est consacré à l'analyse des conditions sous lesquelles un accord d'assurance mutuelle peut résister à l'émergence d'une compagnie privée. Modélisant les principales différences entre mutuelles et compagnies d'assurance, il complète la littérature existante en intégrant les choix d'investissement en capital des compagnies privées. Dans le cas d'agents exposés de manière homogène au risque, cette étude détermine l'unique choix optimal de la compagnie entrante ainsi que les conditions favorisant ou empêchant son apparition.

L'impact sur les accords d'assurance mutuelle d'une possible hétérogénéité sur l'exposition au risque des agents est ensuite abordé dans le deuxième chapitre. Cette analyse permet de présenter l'asymétrie d'information comme une explication possible à l'échec de l'hypothèse de marché complet observé par la littérature empirique. Elle contribue par ailleurs à préciser l'avantage comparatif que possède l'assurance mutuelle en termes d'information.

Enfin, le troisième chapitre prolonge l'analyse des conséquences de l'hétérogénéité face au risque à travers l'étude de l'assurance de long terme. La modélisation de contrats d'assurance mutuelle dynamique permet d'appréhender l'impact de l'aléa moral sur la stabilité des contrats de long terme (face à l'assurance de court terme). L'examen de ces phénomènes met en évidence le rôle des préférences des agents, en termes de prudence et d'aversion au risque, sur la stabilité des contrats d'assurance mutuelle dynamique.

Mots clés : théorie de l'assurance ; mutualisation ; théorie des contrats ; incitations ; asymétrie informationnelle ; anti-sélection ; aléa moral ; faillite ; prudence.

Abstract

This dissertation analyzes mutual insurance and its stability in three different contexts.

The first chapter examines conditions under which an incumbent mutual agreement can resist to the emergence of a private insurance company. We model the main differences between mutual and stock insurers, integrating the investment choices of the insurance company. Focusing on homogeneous agents, we characterize the unique optimal choices of an entrant company and the conditions favoring or preventing its appearance.

The impact of heterogeneity in risk exposure on mutual agreements is studied in chapter 2. This allows putting forward asymmetric information as a possible explanation to the failure of the complete risk pooling highlighted in the empirical literature. We moreover establish that mutual insurance better copes with asymmetric information than insurance companies do.

Eventually, the third chapter extends the study of heterogeneous risk exposure through long term insurance. The use of dynamic contracts allows assessing the impact of moral hazard on the stability of long term mutual agreements (relative to short term insurance). We highlight the role of agents' preferences, in terms of prudence and risk aversion, on the stability of mutual dynamic insurance.

Keywords : insurance ; mutual firms ; mechanism design ; incentives ; asymmetric information ; adverse selection ; moral hazard ; insolvency ; prudence.

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Introduction générale

Le rôle historique des accords d'assurance mutuelle

L'assurance mutuelle représente sans doute le plus ancien instrument mis en place pour se couvrir face au risque. Dès l'antiquité apparaissent des organismes d'entraide mutuelle entre individus exposés à des risques comparables. Les fonds auxquels contribuaient les tailleurs de pierres de la Basse-Egypte au XV$^{\text{ème}}$ siècle avant Jésus-Christ constituent un des exemples les plus anciens de ce type de mécanisme. Développées ensuite - à travers des cotisations mensuelles notamment - par les sociétés bénévoles de la Grèce antique ou les guildes du Moyen-âge, l'assistance réciproque et l'assurance mutuelle permettaient alors de s'assurer contre différents sinistres tels les incendies, les vols ou les inondations.

L'émergence de compagnies d'assurance privées est beaucoup plus récente. On en trouve les prémices dans l'industrie maritime de la Gênes médiévale au XIVème siècle avec l'apparition d'intermédiaires ("third-party insurance") régissant les accords entre "assurés". L'assurance privée s'établit définitivement en Grande Bretagne durant le XVII$^{\text{ème}}$ siècle avec le recours à des capitaux externes pour couvrir le risque d'incendies (le "Fire Office"). Le développement des probabilités et du calcul actuariel durant les XVII$^{\text{ème}}$ et XVIII$^{\text{ème}}$ siècles à travers les travaux de Pascal [57], Huygens [38] ou Deparcieux [16] contribue ensuite à l'émergence de primes d'assurance telles que nous les connaissons aujourd'hui.

Le développement de l'assurance privée ne s'est toutefois pas fait à l'encontre des mécanismes d'assurance mutuelle et les deux formes d'organisation coexistent aujourd'hui. Les compagnies d'assurance ont d'ailleurs elles-mêmes recours à des accords d'assurance mutuelle pour se couvrir contre le risque de défaut à travers les mécanismes de réassurance.

Même sur le marché de l'assurance individuelle, les accords d'assurance mutuelle n'ont pas disparu avec l'émergence de l'assurance privée. Depuis qu'elles coexistent, les deux formes d'organisation ont connu divers succès et aucune d'entre elles ne domine aujourd'hui réellement les marchés de l'assurance. Hansmann (1985) met par exemple en évidence de nombreux changements de forme organisationnelle durant le XX$^{\text{ème}}$ siècle aux Etats-Unis. D'abord organisée sous forme de firmes privées, la plupart des grandes compagnies d'assurance ont fait le choix de la mutualisation au début du XX$^{\text{ème}}$ siècle, la part des mutuelles atteignant même 69% en 1947. À la fin du XX$^{\text{ème}}$ et au début du XXI$^{\text{ème}}$ siècle on a toutefois pu assister à l'effet inverse et de nombreuses mutuelles se sont converties en compagnies privées. Ces phénomènes appelés vagues de mutualisation et de démutualisation posent la question des paramètres gouvernant la préférence pour une forme ou l'autre d'assurance. De même, le contrôle du secteur de l'assurance par les mutuelles santé dans les pays Européens et leur omniprésence dans les pays en développement posent la question des raisons gouvernant la faible part de marché des compagnies privées dans certains pays ou secteurs.

L'objet de la présente thèse est donc d'étudier la stabilité des accords d'assurance mutuelle. On se propose, à travers trois essais, d'étudier dans quelles circonstances un accord d'assurance mutuelle peut être soutenable quand il doit faire face (i) à l'émergence d'une compagnie d'assurance privée, (ii) à une forte hétérogénéité dans l'exposition au risque de ses membres et (iii) à l'écrémage par une compagnie d'assurance de ses membres les moins exposés au risque.

Afin d'étudier plus en avant la stabilité des accords d'assurance mutuelle, il convient de définir précisément les différences existantes entre les deux formes organisationnelles et d'en définir les avantages comparatifs.

Les principales différences entre accords mutuels et compagnies privées en assurance

La principale différence entre une compagnie d'assurance et une organisation fondée sur des accords d'assurance mutuelle (une mutuelle ou une coopérative agricole, par exemple) réside dans la structure de propriété de ces organisations. En tant qu'organisation privée, une compagnie d'assurance est en effet détenue par ses actionnaires alors que, par définition, une mutuelle est propriété de ses assurés. Cette différence a de nombreuses implications, dont trois joueront un rôle particulièrement important dans cette thèse.

Des objectifs différents

La première conséquence directe de ces formes structurelles différentes concerne les objectifs des organisations. Étant détenue par ses actionnaires, l'objectif principal d'une compagnie d'assurance est de garantir la rentabilité du capital investit, c'est-à-dire de créer du profit. Au contraire, en tant que structure détenue par ses membres, un organisme d'assurance mutuelle a pour objectif principal la satisfaction de ses assurés. Ses profits éventuels sont alors réinvestis dans l'organisation ou redistribués.

Cette différence présente par ailleurs une contrepartie tenant à la gestion de l'entreprise. Alors qu'une compagnie d'assurance est gérée par des managers, rémunérés et soumis à la loi du marché, la gestion des accords mutuels est généralement de la responsabilité d'administrateurs bénévoles élus par les adhérents. Ceci implique notamment des " problèmes d'agence ", c'est-à-dire des problèmes managériaux, différents selon le type d'organisation.

Des capitaux d'origine différente

Le recours à des actionnaires dans le cas des compagnies d'assurance conduit par ailleurs à une deuxième différence fondamentale entre les deux formes d'organisation.

Afin de faire face au risque, une compagnie d'assurance fait appel à des capitaux externes alors que les accords mutuels ne reposent que sur des capitaux internes à l'organisation : ceux de ses assurés. Les accords mutuels apparaissent alors comme étant plus exposés au risque agrégé (au niveau du groupe d'assuré), c'est-à-dire au risque macroscopique. En cas de forte perte pour une grande partie des assurés, une compagnie d'assurance pourra, contrairement à une organisation mutuelle, recourir au capital de ses actionnaires.

Une définition du risque différente

L'utilisation de capitaux extérieurs et la recherche de leur rentabilité oblige cependant les compagnies d'assurance à définir de manière complète et précise les risques qu'elles assurent. Ainsi, elles s'engagent avec leurs assurés sur un contrat spécifiant un préfinancement (la prime) et une couverture précise en cas de dommage.

Au contraire, l'objectif collectif d'une organisation d'assurance mutuelle ne lui impose pas de collecter des informations précises sur le risque à assurer. Les cotisations et les remboursements peuvent être ajustés en fonction des réalisations du risque des assurés. Cela permet notamment de construire des accords contingents aux résultats du groupe d'assurés, ce que ne peuvent faire les compagnies d'assurance dont les contrats ne dépendent que des résultats individuels.

Cette différence dans la définition du risque révèle d'autre part l'importance de la notion d'information dans l'étude des formes organisationnelles en assurance. Quand elles possèdent une information complète sur le risque de leurs assurés, les compagnies d'assurance peuvent proposer une assurance actuarielle qui, grâce au recours à des capitaux externe, est toujours plus avantageuse qu'un accord mutuel. Par contre, elles ne peuvent fonctionner sans information, contrairement aux organismes d'assurance mutuelle, qui grâce aux principes de diversification et de mutualisation (exposés dans la section suivante) peuvent proposer un partage de risque à ses assurés, même en l'absence d'information.

Après l'étude des principales différences entre les deux formes d'organisation, il apparaît que les avantages comparatifs des accords mutuels vis-à-vis de l'assurance privée résident dans leur capacité à spécifier des contrats contingents à la réalisation agrégée et à offrir une couverture face au risque sans information. Ces avantages sont cependant contrebalancés par l'exposition au risque macroscopique liée à l'absence de capitaux extérieurs. La neutralité (supposée) au risque des investisseurs offre par ailleurs aux compagnies d'assurance un avantage comparatif dans le support du risque par rapport aux organismes mutuels, propriétés de sociétaires généralement averses au risque.

Les principes de mutualisation et de diversification

À travers la possibilité de construire des contrats contingents au risque agrégé, l'assurance mutuelle repose sur le partage du risque et plus particulièrement sur deux principes généraux, le principe de diversification et le principe de mutualisation.

Le principe de diversification

Le principe de diversification, dont l'intuition apparaît dès les travaux de Bernoulli [5], correspond à l'adage "ne pas mettre tous ses œufs dans le même panier". Selon ce principe la diversification est un moyen efficace de réduction du risque. Formalisé par Rothschild et Stiglitz [61] dans le cadre financier, le principe de diversification énonce qu'*un portefeuille contenant, en proportions égales, n actifs financiers aux rendements identiquement distribués est moins risqué (au sens de la dominance stochastique du second ordre) que tout autre portefeuille contenant ces mêmes actifs.*

Un portefeuille contenant chaque actif en proportion $1/n$ a en particulier la même moyenne mais une variance plus faible que toute autre stratégie. Vrai quelles que soient les préférences des agents (même en dehors du cadre standard de l'utilité espéré), ce mécanisme nécessite cependant que les rendements considérés aient une espérance finie[1].

Réinterprété dans le cadre de l'assurance par Skogh [63], le principe de diversification correspond alors au partage égalitaire du risque, c'est-à-dire à une situation dans laquelle chaque individu prend en charge la perte moyenne du groupe. Ainsi, si les pertes sont identiquement distribuées, la perte moyenne est moins risquée que les pertes individuelles.

[1]Dans le cas contraire, il peut même être plus risqué de partager deux risques dont l'espérance est infinie

Le partage égalitaire

Le partage égalitaire du risque, préconisé par le principe de diversification dans le cas d'une exposition identique au risque, possède par ailleurs des propriétés intéressantes dans le cas des risques non identiquement distribués. En effet, lorsque les individus ne sont pas exposés de façon identique au risque mais ont les mêmes préférences à la von Neumann Morgenstern (représentées par une fonction d'utilité croissante et concave) et les mêmes croyances sur les probabilités de perte, le partage égalitaire apparaît efficace au sens de Pareto.

Notons ici que ce résultat vaut quelle que soit la corrélation entre les risques individuels. Cependant il est très lié à la linéarité en probabilité du critère de von Neuman Morgenstern, mais n'en est pas moins remarquable car il est vérifié dans de très nombreuses configurations de risque, en particulier quand les probabilités subjectives sont différentes des probabilités objectives.

Le partage égalitaire n'est cependant pas nécessairement individuellement rationnel au sens où ne Pareto-domine pas toujours l'état initial (l'autarcie). Pour cela, des hypothèses supplémentaires sont nécessaires. En prolongement le principe de diversification il apparaît en effet que, *si les individus ont une croyance sur les probabilités qui les conduit à penser qu'ils sont tous exposés de façon identique au risque, le partage égalitaire Pareto domine l'état initial.*

Ce résultat met en évidence l'avantage comparatif des accords mutuels énoncés précédemment. En l'absence d'information précise sur le risque (c'est-à-dire dans des cas où l'assurance privée ne peut fonctionner), il suffit que les agents acceptent l'idée qu'il n'y a aucune raison que leurs probabilités de perte diffèrent (présomption d'égalité) pour qu'un partage égalitaire du risque soit mutuellement bénéfique. Un accord d'assurance mutuelle peut alors être implémenté, ce qui n'est pas le cas d'un contrat d'assurance privé.

Le partage égalitaire du risque présente l'avantage de faire disparaître le risque idiosyncratique : la perte supportée ne dépend alors que de la perte moyenne (ou totale) de l'organisation et plus des réalisations individuelles. Cependant, si les risques (i.e. les probabilités de perte) ne sont pas identiquement distribués dans l'organisation, cet outil de réduction du risque peut être contraint par l'aggravation des pertes espérées qu'il cause chez les individus les moins exposés. Partager les pertes avec un individu plus exposé (même si cela réduit la variance) augmente l'espérance des pertes. Le partage égalitaire, qui n'est bien sûr pas l'unique moyen de partager le risque, repose donc fortement sur l'homogénéité des expositions au risque.

Le principe de mutualisation

L'efficacité au sens de Pareto d'autres allocations que le partage égalitaire peut par ailleurs être définie à l'aide du principe de mutualisation.

Introduit par l'article fondateur de Borch [9], le principe de mutualisation (ou principe de mutualité) stipule que, *dans un cadre d'information parfaite, toute allocation efficace au sens de Pareto ne dépend de l'état de la nature qu'à travers le risque agrégé*. Un accord efficace spécifie alors une répartition des richesses identique dans les états de la nature fournissant le même revenu agrégé, quelquesoit la répartition initiale des revenus individuels. Contrairement au principe de diversification, ce second principe repose donc sur l'hypothèse d'information parfaite, mais est vérifié quelque soit l'exposition au risque des individus formant l'accord. Comme indiqué par Gollier [30] et confirmé empiriquement par Townsend [68], ce principe ne semble toutefois pas être vérifié dans les accords réels, et ce pour de multiples raisons, notamment informationnelles.

Structure et enjeux de la thèse

Principaux enjeux de la thèse

Sur la base de ces principes généraux et des différences structurelles précédemment décrites, le principal enjeu de la présente thèse sera l'étude de la stabilité des accords d'assurance mutuelle selon trois axes privilégiés.

Il s'agira dans un premier temps d'étudier dans quelles circonstances de tels accords sont soutenables face à l'assurance privée. Afin d'analyser les conditions sous lesquelles une compagnie d'assurance ne peut pénétrer un marché contrôlé par un accord mutuel, l'accent sera mis, dans une première contribution, sur la notion de capital. On y détaillera notamment les choix d'investissement en capital des compagnies d'assurance en intégrant l'éventualité que celui-ci soit insuffisant, c'est-à-dire la possibilité de faillite.

Les mécanismes présentés supra posent par ailleurs la question de la stabilité des accords d'assurance mutuelle en présence d'hétérogénéité et d'asymétrie informationnelle. Un second enjeu de cette thèse sera donc d'étudier les formes que peuvent prendre ces accords lorsqu'ils concernent des individus exposés de manière différente au risque. On cherchera alors, en utilisant des éléments de la théorie des contrats, à définir les accords permettant d'inciter les individus à révéler l'information qu'ils possèdent sur leur probabilité de sinistre.

Par ailleurs, à cause de leur recherche de rentabilité sur le court terme, les compagnies d'assurance semblent moins à même d'assurer le risque long (notamment en santé) que les accords mutuels. Cette recherche de profit et la nécessité de définir précisément le risque à couvrir peut notamment faire naître pour les assurés le risque d'être classifier "haut risque" par son assureur et donc de se voir facturer une prime élevée. Les accords mutuels semblent pouvoir offrir une couverture contre ce risque dit de classification.

En proposant des assurances intra- et inter- générationnelles et n'étant pas contrainte par des considérations de court terme, l'assurance mutuelle peut en effet offrir un "profil de prime" plus lisse à ses assurés. De tels accords peuvent toutefois être remis en cause par la concurrence d'une compagnie privée qui offrirait aux agents les moins exposés au risque une assurance plus avantageuse. Il apparaît ainsi intéressant d'étudier la stabilité des accords d'assurance mutuelle de long terme face à ces phénomènes "d'écrémage des bons risques".

L'échec du principe de mutualisation

Depuis les travaux initiateurs de Borch ([8] et [9]), l'étude des accords de partage de risque occupe une place importante en économie de l'assurance. Dans son analyse des mécanismes de réassurance, Borch introduit les idées économiques de rationalité et de profit dans la théorie du risque, jusqu'alors essentiellement le fait de statisticiens et d'actuaires. Il définit ainsi un nouveau champ d'étude : la théorie économique de l'assurance. Borch décrit alors un traité de réassurance comme un jeu coopératif de partage de risque. Une telle définition lui permet d'énoncer le principe de mutualisation évoqué précédemment : à l'optimum le montant qu'une compagnie doit payer (en réassurance) ne dépend que du montant total de créances émises par le groupe d'assureurs.

Townsend [68] analyse dans quelle mesure ce principe est vérifié empiriquement par les mécanismes d'assurance informelle des pays en voie de développement. Il utilise pour cela une étude en données de panel sur les consommations des ménages de trois villages indiens. Afin de tester le principe de mutualisation, Townsend régresse la consommation des ménages sur la consommation moyenne du village et le revenu des ménages, parmi d'autres variables de contrôle.

Townsend met ainsi à jour l'existence d'importants mécanismes d'assurance informelle puisque d'une part, la consommation des ménages évolue conjointement (et de manière très significative) à la consommation moyenne et d'autre part, les chocs spécifiques (tels le chômage ou la maladie) n'influencent pas significativement la consommation des ménages. Cependant, les consommations restant par ailleurs corrélées aux revenus des ménages, Townsend rejette l'hypothèse de marché complet, c'est-à-dire le principe de mutualisation.

Depuis lors, une partie de la littérature théorique en assurance analyse les raisons possibles à cet échec du principe de mutualisation. La majeure partie de ces travaux met en avant les limites dans l'engagement des agents pour modéliser de façon endogène les freins à l'assurance complète. L'engagement limité réduirait alors ex-post les transferts contingents. Les agents ayant reçu les revenus les plus élevés durant la période courante et devant alors effectuer des transferts vers les moins chanceux, ont en effet d'importantes incitations à renier leurs engagements. Ainsi, l'engagement à respecter l'accord efficace (l'assurance complète) n'étant pas crédible, des contraintes d'incitation doivent être ajoutées. Un accord sera alors soutenable si aucun des participants n'a d'intérêt à dévier, i.e. si les bénéfices retirés du non respect de l'accord sont plus que compensés par la punition sanctionnant la déviation : la perte des possibilités d'assurance futures (l'exclusion de l'accord).

Ainsi, Coate et Ravallion [13] montrent qu'en prenant en compte cette notion d'engagement limité, l'accord optimal entre deux agents dépend des niveaux de richesse et non plus seulement de la différence de revenu. Les limites dans l'engagement des assurés semblent ainsi pouvoir expliquer la faillite du principe de mutualisation. L'étude de Coate et Ravaillon n'explique cependant pas l'influence des revenus retardés observée par Townsend [68].

Kocherlakota [43] étudie donc le rôle de l'histoire à l'aide d'accords non stationnaires dans le cas de deux ménages symétriques. Une stratégie définit alors, pour toute histoire, le transfert à effectuer. En analysant les équilibres parfaits en sous jeux, Kocherlakota retrouve la corrélation entre consommation présente et revenus passés observée par Townsend : si une contrainte d'incitation sature (c'est-à-dire si les agents sont trop impatients) les transferts d'assurance mutuelle dépendent de l'histoire.

Ligon, Thomas et Worrall [48] complètent les travaux de Kocherlakota en caractérisant le lien entre consommation présente et revenus passés. Dans le cas de deux ménages non symétriques (en termes de préférences), ils montrent que les accords non stationnaires s'apparentent à du quasi-crédit. Le ménage sollicitant de l'aide se voit dans l'obligation de rembourser sa dette dans les états symétriques (lorsque les deux ménages perçoivent le même revenu). En testant leur théorie sur les mêmes données que Townsend [68], Ligon, Thomas et Worrall indiquent que leur modèle d'assurance incomplète explique mieux que l'assurance complète la réponse dynamique de la consommation aux variations de revenu.

Les travaux présentés supra n'analysent cependant pas la formation des accords d'assurance mutuelle, mais uniquement leur stabilité vis-à-vis de l'engagement limité des agents. Pourtant, des études empiriques (cf. Fafchamps et Lund [25] par exemple) ont montré que le partage du risque s'effectue en réalité dans des groupes plus restreints que les villages dans leur intégralité. Il apparaît donc intéressant d'étudier la formation des réseaux d'assurance mutuelle.

Génicot et Ray [29] analysent la constitution des ensembles d'individus partageant les risques, en introduisant la possibilité de déviation collective. Ils supposent cependant qu'un sous-groupe ne dévie que s'il est en mesure de mettre en œuvre un accord soutenable. Aucun des membres de l'accord créé par la déviation, pris individuellement ou collectivement, ne doit avoir intérêt à renier ses (nouveaux) engagements.

Génicot et Ray analysent ainsi la stabilité des groupes d'assurance par une étude récursive des incitations à dévier, en examinant les possibilités d'assurance de chacun des sous-groupes éventuels. Ils démontrent alors que la taille des groupes stables est bornée à la fois inférieurement et supérieurement, la borne inférieure permettant d'éviter les déviations individuelles et la borne supérieure les déviations collectives. Il apparait en effet essentiel, pour que l'accord soit stable, que les possibilités d'assurance mutuelle soient importantes et donc que l'accord rassemble un nombre suffisant de membres. Par ailleurs, le gain marginal de la diversification tendant vers zéro quand le nombre de membres tend vers l'infini, la stabilité par rapport aux déviations collectives requiert un nombre fini de membres.

Concernant toujours la taille optimale des accords informels, un article récent de Bloch, Génicot et Ray [6] définit la notion de fragilité d'un accord d'assurance mutuel. Alors que la stabilité est binaire (un groupe est stable ou non) et concerne tous les états de la nature, Bloch, Génicot et Ray permettent les déviations dans certains états et définissent la *fragilité* comme la probabilité globale qu'une déviation survienne. Les agents anticipent par ailleurs les possibilités de déviation dans leur évaluation de l'accord. Bloch, Génicot et Ray montrent alors que la fragilité croit avec le nombre de membres de l'accord si on considère uniquement les déviations individuelles, mais décroit si les déviations collectives sont prises en compte.

La coexistence des deux formes d'organisation

Si elle permet d'analyser l'échec du principe de mutualisation, l'hypothèse d'engagement limité n'explique pas la coexistence des accords d'assurance mutuelle et de compagnies d'assurance privées.

Une exception notable est le modèle récent de Dubois, Jullien et Magnac [20]. Partant de l'observation qu'il existe dans les pays en voie de développement des institutions pouvant imposer le respect de certains types de contrats, ils proposent un modèle permettant la superposition d'accords informels (avec engagement limité) avec des contrats formels. À partir d'une étude théorique, Dubois, Jullien et Magnac construisent une équation d'Euler qu'ils testent sur données réelles. Cet exercice empirique leur permet de rejeter à la fois le principe de mutualisation et l'hypothèse d'engagement limité, les niveaux de consommation passés expliquant à la fois les niveaux courants de consommation et de revenu. Ils mettent par ailleurs en évidence le rôle des contrats formels dans la variance des revenus intra-village. Ainsi, l'échec du principe de mutualisation dans les accords mutuels d'assurance semble pouvoir être expliqué à la fois par l'engagement limité et la coexistence de contrats formels.

Les raisons de la coexistence de deux formes d'organisation ne sont toutefois pas abordées dans les articles précédemment cités. Si des problèmes d'information sont avancés par Dubois, Jullien et Magnac notamment, ils ne sont pas modélisés.

La littérature relative à la coexistence entre accords mutuels et assurance privée met par ailleurs en évidence d'autres explications que l'asymétrie informationnelle.

Ainsi, Mayers et Smith [49] soutiennent que la coexistence des deux formes provient de leur capacité à gérer différents problèmes d'agence. Les mutuelles existeraient grâce à leur capacité à internaliser les conflits d'intérêt pouvant naîtrent entre assurés et actionnaires d'une compagnie privée. Les dirigeants des compagnies d'assurance peuvent en effet être amenés à privilégier le versement de dividende au détriment de l'intérêt des assurés. Les deux groupes coexistant dans le cas d'accords mutuels, ce type de conflit "assurés-actionnaires" est évité. Pour expliquer la coexistence des deux formes, Mayers et Smith affirment que cet avantage comparatif est balancé par celui que possèdent les compagnies privées dans le contrôle de leurs manageurs. En utilisant des mécanismes incitatifs comme les prises de participation ou les stock options, les compagnies privées semblent en effet mieux armées pour faire face au conflit entre actionnaires et manageurs. Les compagnies privées semblent donc devoir dominer les lignes d'assurance nécessitant le plus de précaution managériale, et l'assurance mutuelle apparaît comme étant plus à même de couvrir des risques de long-terme, soumis à davantage de conflits entre actionnaires et assurés.

En complément de ces arguments managériaux, un pan de la littérature explique la coexistence des deux formes d'organisation par la nature participative des accords mutuels. Smith et Stutzer [64] suggèrent ainsi que les mutuelles attirent les individus les moins exposés au risque qui veulent se signaler, en montrant qu'ils sont prêts à supporter le risque macroscopique. En utilisant un modèle à la Rothschild et Stiglitz [62], Smith et Stutzer suggèrent donc que compagnies d'assurance et mutuelles s'adressent à des types d'agents différents qui s'auto-sélectionnent en choisissant l'organisation par laquelle ils veulent être assurés.

Parallèlement, si le risque à assurer peut être décomposé en une partie diversifiable (i.e. idiosyncratique) et une partie non-diversifiable, Doherty et Dionne [18] montrent qu'il est optimal de combiner les deux formes d'assurance. Il semble en effet que le risque non-diversifiable puisse être trop coûteux à assurer pour une compagnie privée, auquel cas celle-ci laisse ses assurés avec un risque résiduel. Ces derniers peuvent alors utiliser un accord mutuel pour couvrir la partie du risque non assuré par la compagnie.

Afin de compléter ces travaux, Laux et Muermann [46] étudient le choix optimal de capital de compagnies soumises à des conflits entre actionnaires et manageurs et introduisent la possibilité de faillite. En l'absence de compétition entre compagnies d'assurance et mutuelles, ils démontrent tout d'abord qu'il est optimal pour les assurés de transférer la richesse des états dans lesquels leur compagnie est solvable vers ceux où elle ne l'est pas. Ainsi, les agents diminuent la partie résiduelle du risque qu'ils supportent. Par ailleurs, en endogénéisant le choix du capital, Laux et Muermann montrent que l'incitation à augmenter le nombre d'assurés est plus importante pour les mutuelles que pour les compagnies d'assurances.

S'ils permettent d'appréhender les avantages comparatifs des mutuelles et des compagnies d'assurance, les travaux précédemment exposés considèrent les deux types d'organisations comme des entités indépendantes et ne modélisent pas la compétition qui peut exister entre les deux formes d'assurance.

Fagart, Fombaron et Jeleva [26] modélisent les interactions entre mutuelles et compagnies d'assurance et étudient la compétition que les deux formes peuvent se livrer pour attirer les assurés. Elles démontrent que l'utilité espérée d'un agent dépend alors de la taille de l'organisation à laquelle il appartient. L'existence de mutuelles modifie donc le comportement optimal des compagnies d'assurance à travers ces "effets de réseau". Par ailleurs, Fagart, Fombaron et Jeleva supposant le stock de capital fixé de manière exogène, les deux formes organisationnelles peuvent coexister à l'équilibre de leur modèle.

La coexistence des deux formes d'organisation en assurance et la compétition qui peut exister entres elles fait également l'objet de plusieurs études empiriques.

Un premier champ d'étude de cette littérature concerne les effets de réseau qui viennent d'être évoqués. Nekby [54] confirme par exemple le résultat théorique de Laux et Muermann [46] et montre, en utilisant des données historiques suédoises, qu'une mutuelle est en moyenne significativement plus grande qu'une compagnie d'assurance. L'étude de Nekby est par ailleurs remarquable car elle porte sur une période (1902-1910) pendant laquelle aucune régulation ne contraignait le fonctionnement de chacune des formes d'organisation. Ce résultat peut par ailleurs être relié à ceux de Viswanathan et Cummins [70] et d'Erhemjamts et Leverty [24] sur les récentes vagues de démutualisation aux États-Unis. Ces deux études mettent en effet en évidence le rôle de l'augmentation de la compétition sur ces changements de formes organisationnelles. La compétition, en diminuant le nombre d'assurés, inciterait les mutuelles à se transformer en compagnies privées, conformément aux résultats précédents.

Une partie de la littérature empirique analyse parallèlement l'effet du risque sur le choix de la forme d'organisation. Lamm-Tennant et Starks [45] démontrent ainsi, en mesurant le risque par la variance du ratio de perte, que les compagnies privées assurent généralement des activités plus risquées que les organisations mutuelles. Cette étude confirme ainsi le résultat théorique de Mayers et Smith [49] si on considère que les lignes d'assurance les plus risquées nécessitent une plus grande prudence managériale. Dans une étude plus récente, Mayers et Smith [50] confirment d'ailleurs ce lien entre risque et formes d'organisation, en analysant quatre-vingt-dix-huit conversions de mutuelles en compagnies privées aux États-Unis entre 1920 et 1990. Ils vérifient, en utilisant une analyse moyenne-variance puis une régression probit, que la probabilité de conversion est d'autant plus grande que le risque couvert est grand.

En complément de ces études sur la compétition entre mutuelles et compagnies privées, une analyse de la complémentarité des deux formes d'organisation s'est développée dans le cas de l'assurance sociale (qui peut être considérée comme un accord d'assurance mutuelle à l'échelle de la société).

Casamatta, Cremer et Pestiau [10] étudient ainsi le cas d'une police d'assurance privée proposée en complément d'une assurance sociale ayant un objectif en partie redistributif. Les agents non satisfaits du caractère partiel de l'assurance sociale (choisie par vote) peuvent alors la compléter par une assurance privée. Casamatta, Cremer et Pestiau montrent que l'introduction de l'assurance privée diminue la générosité de l'assurance sociale mais peut tout de même bénéficier aux plus pauvres (bien qu'ils ne l'achètent pas). Cet effet quelque peu contre-intuitif provient du fait que Casamatta, Cremer et Pestiau supposent que la couverture d'assurance est la seule source de revenu en cas de dommage. Ainsi, si la fonction d'utilité est suffisamment concave, les individus les plus riches sont prêts à payer une cotisation plus élevée pour obtenir une couverture plus importante.

En se basant sur l'expérience du Chili, Goulão [33] complète le débat en introduisant la notion d'assurance sociale volontaire. Un individu a dans ce cas la possibilité de participer ou non à l'assurance sociale en complément de l'assurance privée. Goulão analyse le cas d'agents exposés de manière hétérogène au risque, de telle sorte que les compagnies d'assurance ne peuvent offrir aux agents les moins exposés qu'une couverture partielle (cf. Rothschild et Stiglitz [62]), créant ainsi un spectre pour l'assurance sociale. Un des principaux résultats de l'article est alors la soutenabilité de l'assurance sociale malgré son caractère volontaire : il y aura toujours des individus souhaitant ajouter cette assurance sociale à leur assurance privée. Par ailleurs, l'existence de l'assurance sociale peut augmenter l'efficacité du marché privée en permettant aux agents de s'auto-sélectionner. L'information révélée par la participation à l'assurance sociale peut être utilisée par le marché pour construire les contrats et améliorer la couverture offerte aux individus les moins exposés au risque.

Goulão suppose cependant que l'assurance sociale ne tient pas compte de l'hétérogénéité des agents en termes d'exposition au risque. Dans ses travaux, la couverture et la prime sociale dépendent uniquement du revenu et non du risque individuel.

L'hétérogénéité des agents face au risque

D'une manière plus générale, l'hétérogénéité de l'exposition au risque des agents n'est que très peu prise en compte dans la littérature relative aux accords d'assurance mutuelle.

Le comportement des compagnies d'assurance face au problème d'asymétrie d'information sur ce type d'hétérogénéité est pourtant bien documenté (cf. Rothschild et Stiglitz [62], Stiglitz [65] ou Chade et Schlee [11]).

Le résultat de base concerne le cas de compagnies d'assurance en concurrence faisant face à deux types d'individus : les "haut risque" et les "bas risque". En présence d'asymétrie informationnelle, pour inciter les agents à révéler leur exposition au risque, une compagnie d'assurance doit alors proposer un contrat séparateur spécifiant pour les "haut risque" une assurance complète au prix actuariel et pour les "bas risque" une assurance partielle au prix actuariel.

Dans le cadre de l'assurance mutuelle, Doepke et Townsend [19] introduisent une potentielle hétérogénéité (ex-post) dans l'exposition au risque en modélisant la possibilité d'effectuer des actions (efforts) modifiant la distribution des revenus. Doepke et Townsend étudient cependant les contrats incitatifs de telle sorte que les agents effectuent tous (ex-post) l'effort recommandé par le principal et possèdent ainsi la même exposition au risque.

Ligon et Thistle [47] considèrent au contraire le cas d'agents hétérogènes ex-ante dans leur étude de la taille optimale des accords d'assurance mutuelle. Dans le cadre d'accords spécifiant nécessairement un partage égalitaire du risque, leur principale contribution consiste à montrer que toutes les configurations stables de mutuelles sont séparatrices, les "haut risque" et "bas risque" se regroupant dans des accords d'assurance mutuelle différents.

Afin d'analyser le design des mécanismes de partage optimaux en présence d'hétérogénéité, il semble nécessaire de se tourner vers la littérature relative au micro-crédit (qui peut s'apparenter à de l'assurance mutuelle contre le risque de défaut).

Towsend [69] étudie ainsi le rôle de l'aléa moral dans le financement d'un projet avec responsabilité jointe ("joint liability"). Il met alors en évidence la contingence des accords avec des variables exogènes au groupe notamment macroéconomique. En confrontant les différents modèles théoriques existants avec des données issues de l'économie réelle, Townsend relève notamment que le modèle pertinent dépend du degré de développement de la zone géographique concernée.

Concernant le problème complémentaire de la sélection contraire, Armendariz et Gollier [3] considèrent le cas d'une banque cherchant à fixer le taux d'intérêt optimal offert à un groupe d'emprunteurs s'assurant mutuellement. En considérant deux groupes d'emprunteurs, les "risqués" et les "sûrs", Armendariz et Gollier montrent que la mutualisation du risque entre les emprunteurs permet de réduire le taux d'intérêt.

Le risque de classification

Même en présence d'information symétrique, l'hétérogénéité des expositions au risque peut être problématique, notamment dans le cadre de l'assurance de long terme. L'existence d'une telle hétérogénéité amène en effet les compagnies d'assurance à proposer des contrats dépendants du type de risque de chaque agent. Les individus les plus exposés au risque se voient alors offrir un contrat d'assurance stipulant une prime élevée et parfois inabordable. Cela créé ainsi pour les agents un risque d'être classifié "haut risque" par son assureur, appelé *risque de classification.*

La littérature, notamment en assurance santé, s'est attachée à construire des mécanismes d'assurance contre ce risque de classification. Pour ce faire, Pauly, Kunreuther et Hirth [58] proposent par exemple un contrat d'assurance dynamique spécifiant un schéma de primes décroissantes dans le temps, appelé *assurance avec renouvellement garanti* ("guaranteed renewable insurance"). Ils construisent un profil temporel de primes tel que la prime actuelle ne dépasse jamais les dépenses futures espérées. Fricks [27] leur oppose cependant que ce type de solution implique de fortes dépenses en début de vie, et qu'il est donc difficile à implémenter en réalité. S'ils sont trop impatients ou s'ils rencontrent des contraintes de crédit, les agents achètent au mieux une assurance avec renouvellement partiellement garanti.

Cochrane [14] propose une solution alternative reposant sur des indemnités de rupture appelées "severance payments". Un agent devenant "haut risque" reçoit alors un transfert forfaitaire (financé par les indemnités de rupture) égal à l'augmentation de sa prime. Le système d'indemnités de rupture compense ainsi les évolutions de primes, et permet à chaque assuré d'acheter une assurance à son prix actuariel. Pauly, Nickel et Kunreuther [59] notent cependant que cette solution ne peut implémentée qu'en présence d'un marché du crédit parfait. Par ailleurs, Hendel et Lizzeri [36] insistent sur le fait que de tels contrats ne peuvent exister en réalité pour des raisons légales.

Hendel et Lizzeri [36] proposent alors une solution reposant sur la mutualisation inter- et intra-générationnelle. Ils construisent pour cela un modèle à deux périodes dans lequel (i) en première période, les agents sont homogènes et ont la possibilité de transférer de la richesse vers la seconde période via le paiement d'une prime plus élevée (on parlera alors de prépaiement) et (ii) en seconde période - pendant laquelle apparait le risque de classification - les agents les moins exposés peuvent subventionner les "plus hauts risque". Hendel et Lizzeri prennent par ailleurs en compte le phénomène d'écrémage c'est-à-dire la coexistence de l'assurance de long terme avec des contrats (spot) de court terme. Ainsi les agents faiblement exposés au risque en seconde période peuvent être incités à quitter l'accord de long terme pour rejoindre un assureur leur proposant un contrat plus avantageux sur une période. Hendel et Lizzeri montrent alors que le recours au prépaiement réduit à la fois le risque de classification et l'écrémage des "bon risque", retrouvant ainsi une observation empirique qui veut que les contrats d'assurance-vie spécifiant le plus de prépaiement connaissent aussi le plus faible taux d'annulation ("lapsation rate").

En offrant la même assurance à des individus exposés de manière différente au risque, le mécanisme proposé par Hendel et Lizzeri pose toutefois un problème d'aléa moral. En effet, en supposant que les agents peuvent effectuer en première période un effort réduisant leur exposition au risque de seconde période, un contrat spécifiant la même couverture pour différents types de risque tend à réduire cet effort, appelé effort de prévention primaire.

Ce type d'effort est modélisé par Nishimura [56] dans son étude des contrats d'assurance dynamique en présence d'aléa moral. Cette extension du modèle d'Hendel et Lizzeri a pour principal effet de rendre l'apparition de prépaiement dépendante de l'aversion au risque des agents et de l'efficacité de la prévention (c'est-à-dire de l'influence de l'effort sur l'exposition au risque). Nishimura montre de plus que la modélisation du marché du crédit (i.e. la possibilité d'épargner) n'influence pas ce résultat, alors qu'elle conduit dans tous les modèles présentés supra à un lissage complet (inter- et intra-temporelle) de la consommation.

Structure de la thèse

La présente thèse a pour objectif de compléter la littérature précédemment exposée autour de trois axes principaux :

- l'introduction du choix de capital des compagnies d'assurance dans l'analyse de la compétition entre les deux formes d'organisation,
- l'étude des accords d'assurance mutuelle incitatifs dans le cas d'agents exposés de manière hétérogène au risque,
- l'analyse de l'impact de l'aléa moral sur le phénomène d'écrémage, i.e. la stabilité des accords de long terme face à des contrats spots.

Chacun de ces axes est l'objet d'une contribution théorique présentée dans un chapitre de cette thèse.

Un premier chapitre intitulé **On the Emergence of Private Insurance in Presence of Mutual Agreements** présente l'étude des circonstances dans lesquelles un accord d'assurance mutuelle peut être soutenable, c'est-à-dire des conditions sous lesquelles une compagnie d'assurance ne peut pas pénétrer un marché contrôlé par un tel accord.

Ce chapitre complète la littérature existante en intégrant les choix d'investissement en capital des compagnies d'assurance à l'étude de la compétition entre les deux formes d'organisation. Cette adjonction permet notamment de prendre en compte la possibilité de faillite des compagnies d'assurance. La stabilité de l'assurance mutuelle est alors abordée en étudiant les choix optimaux (prime et stock de capital) d'une compagnie d'assurance cherchant à pénétrer un marché contrôlé par des accords mutuels. Dans ce chapitre, les agents sont supposés homogènes et l'accord mutuel optimal correspond au partage égalitaire. En définissant les paramètres influençant le profit optimal d'une telle compagnie, ce travail permet donc de préciser les configurations dans lesquelles un accord mutuel ne peut être contesté par une compagnie privée.

Cette analyse théorique confirme par ailleurs la plupart des résultats empiriques précédemment décrits. Il apparaît tout d'abord que la possibilité pour une compagnie d'assurance de concurrencer un accord mutuel est d'autant plus faible que la taille de la population assurée par l'accord mutuel est grande. Le modèle présenté dans ce chapitre semble ainsi cohérent avec les observations empiriques de Nekby [54], Viswanathan et Cummins [70] et Erhemjamts et Leverty [24]. Par ailleurs, conformément aux résultats de Lamm-Tenant et Starks [45] et Mayers et Smith [50], une augmentation du risque (mesurée par une augmentation de la variance des revenus individuels) tend alors à diminuer la stabilité des accords mutuels, en augmentant le profit espéré du potentiel entrant. Enfin, ce travail met en évidence le rôle du coût du capital et de l'aversion au risque des agents sur la stabilité de l'assurance mutuelle. Il apparaît en effet dans ce chapitre que la possibilité pour une compagnie d'assurance de concurrencer un accord mutuel est d'autant plus grande que le coût du capital est faible et l'aversion au risque des agents importante.

Le deuxième chapitre, intitulé **Mutual insurance with asymmetric information : The case of adverse selection** est issu d'un travail réalisé en collaboration avec Dominique Henriet, et complète la littérature en étudiant comment les accords d'assurance mutuelle peuvent résoudre le problème de sélection contraire en présence d'hétérogénéité face au risque.

En utilisant les outils de la théorie des contrats, cette contribution caractérise l'accord mutuel devant être mis en place afin que des agents exposés différemment à un même risque soient incités à révéler l'information qu'ils possèdent sur leur probabilité de sinistre. On se place pour cela dans le cadre simple d'un accord entre deux agents pouvant être exposés de deux manières différentes au risque ("haut risque" ou "bas risque").

Une telle analyse permet de présenter l'asymétrie d'information comme une explication alternative à la faillite du principe de mutualisation. En effet, en présence d'information symétrique, le principe de mutualisation est soutenable malgré l'hétérogénéité des agents : à l'optimum, le revenu des agents ne dépend que de la réalisation agrégée. Ce n'est toutefois plus le cas si on considère que l'information relative à l'exposition au risque est privée. Afin d'inciter les agents à révéler cette information, il est alors nécessaire de rendre les contrats contingents aux réalisations individuelles. Le mécanisme incitatif se fait donc au détriment du principe de la mutualisation.

Par ailleurs, cette contribution confirme le relatif avantage comparatif que possèdent les accords mutuels en termes d'information. Alors qu'il a été montré que les contrats des compagnies d'assurance perdent nécessairement en efficacité en présence d'asymétrie informationnelle (cf. Rothschild et Stiglitz [62] et Stiglitz [65] notamment), il apparait dans ce chapitre que l'asymétrie d'information ne modifie pas nécessairement les accords mutuels. Plus précisément, si les agents sont suffisamment averses au risque ou si la différence entre les deux types d'agents est suffisamment faible, le partage égalitaire du risque sera optimal malgré l'asymétrie d'information.

Enfin, le troisième chapitre **Moral hazard in dynamic insurance, Classification Risk and Prepayment** a pour objet l'analyse du risque de classification en présence d'aléa moral. Les solutions proposées par la littérature pour résoudre le problème du risque de classification présentent en effet pour la plupart un problème d'incitation à l'effort de prévention primaire. L'objectif de cette contribution est donc d'analyser dans quelle mesure le rétablissement de cette incitation créé de nouveau un risque de classification. Le modèle présenté dans ce chapitre prend par ailleurs en compte la possibilité de prépaiement des primes et les phénomènes d'écrémage.

Ce travail met en évidence le rôle de la prudence et de l'aversion au risque dans la détermination des efforts de prévention primaire et du degré de prépaiement. Ainsi, si le coefficient absolu de prudence est supérieur (respectivement inférieur) à deux fois le coefficient absolu d'aversion au risque, la prise en compte de l'aléa moral augmente (resp. diminue) le prépaiement et le risque de classification. Ce résultat met en évidence l'arbitrage fait par les agents face au risque de classification, entre effort de prévention et prépaiement des primes, un agent plus prudent qu'averse au risque préférant alors prévoir (à travers des transferts intertemporels de richesse) que prévenir (à travers un effort de prévention primaire).

Dans le cas d'agents aux préférences CRRA (i.e. dont le coefficient d'aversion relatif au risque est constant) il apparait par ailleurs qu'un accord d'assurance mutuelle est d'autant plus stable par rapport au phénomène d'écrémage qu'il concerne des agents prévoyants (i.e. des agents préférant le prépaiement à l'effort). Enfin il apparait possible de retrouver pour certaines classes de fonctions d'utilité le fait stylisé exposé par Hendel et Lizzeri [36], selon lequel les contrats spécifiant le plus de prépaiement sont aussi les moins exposés à l'écrémage.

Chapter 1

On the Emergence of Private Insurance in Presence of Mutual Agreements[1]

1.1 Introduction

Historically, mutual agreements have been the first mean used to cope with risk. Starting from benevolent societies in the ancient Greece or guilds in the Middle-Age, reciprocal help and mutual assistance have been first used by people to be insured against various risks as fire, robbery or floods. The emergence of private insurance companies is very posterior. Starting in medieval Genoa during the 14th century with third-party insurance in shipping industry, private insurance definitively arises in Great Britain with fire insurance in the 17th century and the use of external capital. Since, both organizational forms experienced various success and none of them really dominates insurance markets. For example, as stated in Hansmann [35], many changes in organizational forms happened in the United States during the 20st century. First organized as private organizations, many of the largest insurance companies choose to mutualize in the earlier part of the century. The share of mutual firms in life insurance even hit 69 percent in 1947.

[1] This chapter reviews a work registered as *MPRA Paper* n°5821 submitted to publication. A previous version has been circulated as GREQAM working paper n°2006-46 under the title "How Can Insurance Companies Compete With Mutual Insurers ? The Role of Commitment."

However, during the second part of the 20st century and the beginning of the 21st, the reverse effect arisen and many mutual firms have converted to the stock form. These phenomenons called mutualization and demutualization waves, rise the issue of the parameters driving the preference for one form or the other. Similarly, as mutual forms control health insurance in most European countries and as risk is largely insured with mutual agreements in developing countries it seems worthwhile to study why insurance companies have very low market shares in some countries or sectors.

The main objective of this chapter is to analyze the influence of the pre-existence of mutual firms on the choices of an entrant insurance company when optimal behavior consists in decisions about both offered coverage and capital stock. We analyze more precisely the impact of the existence of mutual firms as individuals outside option on the optimal profit of an unique entrant insurance company. We focus on the effects of parameters as the cost of capital, the distribution of income, the degree of risk aversion and the size of the population. In this way we are able to analyze how and when an insurance company may attract mutual firms policyholders and to determine which variables make or not an insurance company enter the market. However in this paper we do not consider the entry of further insurance companies and are thus unable to study the impact of openness to competition in a regulated market. Still, by studying when a company can not rule out mutual agreements our work analyzes when mutual agreements may be sustainable and why some market remain reserved to such arrangements even without any regulation.

Our analysis also appears to be useful in the determination of the capital stock needed by an insurance company as it defines the optimal capital required to insure a given risk when insurers are limited liability companies.

To do so, we build a model that captures the main features distinguishing mutual and stock insurers, namely : (i) a difference in the ownership structure : while insurance companies are owned by their shareholders, mutual firms belong to their policyholders, (ii) a difference in the objective of the organization : whereas insurance companies aim to maximize return on invested capital, mutual firms theoretically maximize its members satisfaction, and (iii) a difference in the definition of risk : stock firms have to precisely define at stake risks to contract on a fixed premia when mutual ones can define risk ex-post as they systematically adjust offered premia a posteriori. These three differences imply several trade-offs between the two organizational forms. Firstly, as shareholders are assumed to be risk-neutral, while policyholders are risk adverse, the insurance company appears to have a comparative advantage in bearing risk. However, raising capital externally is costly and as shareholders are profit seeker, conflicts may arise between shareholders and policyholders. As this paper introduces in the discussion the investment choice of the insurance company, the last difference will have an important role in our setting. Indeed, although external capital is highly useful when aggregate loss is high, it may become insufficient to honor the specific contract the firm commits on and the company may become insolvent. Like most of the papers on this topic we then assume that agents are perfectly rational and thus take into account the probability of insolvency when making their choices. Therefore an individual may not ever wish an increase in coverage as it also increases the insolvency probability of the insurance company.

Under such considerations, we characterize in this chapter the optimal choice of coverage and capital investment of a single entrant company that faces an incumbent mutual firm, and show it is unique. In doing so we are able to determine the conditions under which this equilibrium gives a positive expected profit, that is to state when an insurance firm can enter the market. In analyzing these conditions, this paper provides interesting comparative statics, either based on analytical results or simulations. This way we show that a decrease in the cost of capital raises the optimal capital stock, lowers optimal proposed coverage and thus rises the likelihood for a stock firm to be set up. We also prove in this paper that when a distribution of aggregate income dominates another one in the sense of first order stochastic dominance, the optimal offered coverage increases. Another result of interest is the fact that a higher individual degree of risk aversion increases optimal capital reserves, decreases optimal coverage and that those two forces result in an increase in the optimal profit. Simulations on the influence of the insured population size then allow to state that, when risks are independent, an increase in the number of policyholders raises the optimal offered coverage and lowers the possibility for a stock company to emerge. Lastly, we prove in this paper that the opportunity for an insurance firm to enter the market is higher when individual risk is high, as an increase in the variance of income increases optimal capital reserves and decreases optimal coverage. Those two last results are consistent with the findings of previous empirical works either on demutualization or on the difference between the two organizational forms.

We briefly discuss the relationship of the paper with the most closely related literature. This paper fits into the literature on organizational form in insurance that first tries to explain the coexistence of mutual firms with insurance companies. Focusing mainly on the difference in the ownership structure, Mayers and Smith [49] argue that the two organizational forms coexist because each ownership structure has a comparative advantage in preventing different types of agency problems (mutual firms prevent for conflict between shareholders and policyholders but have less incentive to control their managers). Alternatively, Smith and Stutzer [64] and Doherty and Dionne [18] take into account the additional feature that policyholders of mutual firms bear the aggregate risk (that is that mutual firms – contrary to stock companies – offer participating policies) to explain this coexistence arguing that stock and mutual firms insure different kind of individuals or different kind of risks [2].

[2] Doherty and Dionne [18] prove that, if covered risks are decomposable into diversifiable (idiosyncratic) and non-diversifiable parts, it is optimal, given participating nature of mutual firms

Ours is not the first paper to includes the possibility of insolvency. Focusing on the solvability regulation, Rees, Gravelle and Wambach [60] show that, in absence of mutual firms, it is optimal for the insurance companies to hold enough capital to avoid insolvency when total losses are bounded. However, if it is not always feasible to escape bankruptcy or if the market is not frictionless, this result does not hold.

Laux and Muermann [46] study optimal choices of mutual and stock insurers when there are frictions and more precisely when there exist conflicts between managers and owners. They first show that, without any competition between stock and mutual firms, it is optimal for policyholders to transfer wealth between solvency and insolvency states. Making capital choice endogenous, they show that capital stock and premia are both decreasing with governance problems and increasing with competition. Finally they prove that the incentive to increase the number of policyholders is higher for mutual firms. They however consider stock and mutual firms as independent entities that do not compete to attract policyholders.

On the contrary when analyzing the impact of mutual firms on the insurance market, Fagart, Fombaron and Jeleva [26] model the interactions between insurance companies and mutual firms but do not study optimal capital choice. They show that the expected utility of the consumers depends on the size of the organization they belong to and thus that the existence of mutual firms modifies optimal behavior of insurance companies, when it only consists of offered premia. In their paper, the network effects lead to multiple equilibria. Moreover as the insurance company is limited in size because of fixed capital stock, the two organizational forms may coexist at the equilibrium. In this paper however, we endogenize the choice of capital and define the optimal choice of the company in a way that allows for more comparative statics results. We are then able to better analyze the emergence

policies, to combine insurance firms (non-participating) coverage with mutual risk sharing arrangements. On the same direction, Smith and Stutzer [64] show, using a variant of adverse selection model of Rothschild and Stiglitz [62], that because of their participating nature, mutual firms attract low risk individuals who want to signal their type.

of private insurance and to characterize the setting (depending on the risk distribution and the risk aversion among other things) in which each organizational form dominates. Compared to Fagart, Fombaron and Jeleva [26] this is done at the price of considering a unique stock firm. Moreover, with endogenous stock of capital, the coexistance of both a mutual firm and an insurance company is impossible at the equilibrium. If it enters the market, an insurance firm finds always profitable to hold enough capital to insurance the whole market. In this way our model explains why some markets are dominated by one organizational form, and gives a rationale for mutualization and demutualization waves.

Our paper contributes to the literature on insurance forms by studying both the interactions between the two organizational forms and the investment choices of insurance companies.

The rest of the chapter is structured in the following way. We present the model in Section 2 before characterizing the optimum and its implication on firms participation (in Section 3). Comparative statics either based on analytical results or simulations are provided in Section 4 and compared to previous empirical findings in Section 5. Our conclusion and directions for future research are outlined in Section 6.

1.2 The Model

1.2.1 General Assumptions and Notations

We consider n identical risk averse individuals with increasing and concave utility function $u(.)$ that satisfies the Inada conditions. Each agents receive random revenue $\widetilde{\omega}_i$, $i = 1, ..., n$, the $\widetilde{\omega}_i$s being independent[3] and identically distributed. We assume then that aggregate revenue in the economy called $\widetilde{\omega} \equiv \sum_{i=1}^{n} \widetilde{\omega}_i$ is distributed according to some cumulative distribution function $F(.)$ with density $f(.)$. This random variable may be interpreted as total crop or a sum of revenues adjusted for uncertain health spending for example.

1.2.2 The Insurance Process

As agents are risk averse, they want to be insured against risks of changes in revenue and we consider that they face two kinds of organizations to do so.

- They originally share risk thanks to a mutual agreement. In our static framework, such an agreement corresponds to a sharing rule of the aggregate revenue and may therefore be interpreted as a cooperative or a tontine fund. Indeed, whatever the coverage specified, a mutual firm being collectively owned by its policyholders, they receive any extra profit at the end of the period, such that the whole revenue is shared. Following Borch [9], Eeckhoudt and Gollier [21] and Fagart, Fombaron and Jeleva [26] we then have that the optimal sharing rule is characterized by following proposition.

[3] The assumption of independence is not necessary for our main results to hold. However, to relax it we need to model a specific form of correlation. For example, as shown in Henry [37], our analysis is likely to remain accurate for a f-modulated stochastic dependence

Proposition 1.1 *When individual risks are independent and identically distributed, a mutual aggreement optimally provides equal sharing of resource such that each policyholder of a mutual firm with n members gets $u\left(\frac{\omega}{n}\right)$ whatever the state of the world. Moreover, as shown in Fagart, Fombaron and Jeleva [26], the expected utility of mutual policyholders is increasing with the number of people in such an agreement.*
Proof*: See Appendix*

- They can choose to subscribe to a policy in an entrant insurance company. This firm is owned by shareholders that invest in the insurance market a capital stock K at the beginning of the process (i.e. before the realization of the $\widetilde{\omega}_i$s is known) and gets the profit of the company Π at the end (that is after having indemnified the policyholders). They however face a discount factor δ that represents the opportunity cost of capital[4]. We moreover assume that this company is a limited liability firm and can not borrow in top of this capital investment.

  When insuring a random revenue $\widetilde{x}$, the insurer therefore solves the following program that determines, for a given capital stock, the benefits $\pi(x)$ it earns in each state x to maximize its expected profit, under the limited liability and participation constraints:

$$\max_{\pi(x)} \quad \left\{\delta.\int_{-\infty}^{+\infty}\pi(s)f\left(s\right)ds-K\right\} \tag{1.1}$$

$$\text{s.t.} \quad \begin{cases} \pi(x)\geq -K\ \forall x \\ \displaystyle\int_{-\infty}^{+\infty}\left[u\left(s+K-\pi(s)\right)\right]f\left(s\right)ds\geq\int_{-\infty}^{+\infty}u(s)f\left(s\right)ds \end{cases}$$

[4]If capital has no opportunity cost or if there is no discount factor, the company has an incentive to accumulate an infinite amount of capital and thereby avoid bankruptcy.

Letting the Lagrange multipliers be $\nu(x)f(x)$ and Γ respectively, the first order condition writes $1+\nu(x)-\Gamma u'\left(x+K-\pi(x)\right)=0$. Therefore, when the insurance company is solvent (i.e. when $\nu(x)=0$) it optimally offers a fixed coverage to its policyholders (as then $u'\left(x+K-\pi(x)\right)=1/\Gamma$ is constant). Conversely, when the limited liability constraint binds, the insurer is forced to expend its capital stock.

In our context this means that a company that insures the entire population goes bankrupt when $\omega < n.y-K$, where y represents the fixed coverage offered by the company when it is solvent. Then, the probability of insolvency is equal to $F(n.y-K)$. In cases of bankruptcy, because of limited liability, the firm has to shares its whole resources (premia plus capital) among its policyholders. Therefore each policyholder of a company that insures the n agents gets $u\left(\dfrac{\omega+K}{n}\right)$ (which is less than $u(y)$ when $\omega < n.y-K$) [5]. As we assume that policyholders fully anticipate this probability of insolvency, the expected utility of an individual insured with all the others in the entrant insurance company is :

$$U(y,K)\equiv[1-F(n.y-K)].u(y)+\int_{-\infty}^{n.y-K}u\left(\frac{\omega+K}{n}\right)f\left(\omega\right)d\omega \tag{1.2}$$

Remark 1.1 *The expected utility of an individual insured in the entrant stock firm is increasing in both the offered coverage and the company capital stock.*

[5] The main results and properties of our analysis remain unchanged even if policyholders only have priority on total premia and a portion $0\leqslant\lambda\leqslant 1$ of the capital stock. In this case it can be shown that it is all the more difficult for a insurance company to enter the market as λ is low, that is as shareholders have priority on a large part of the capital stock

The reader should note that this contract can easily be related to usual insurance contracts that implies an indemnity and a premium. Considering $\widetilde{\omega}_i$ as composed by a certain revenue $\overline{R}$ minus a random positive loss $\widetilde{l}_i$, y can then be defined as $y = \overline{R} - \widetilde{l}_i - \pi + i\left(\widetilde{l}_i\right)$, where π represents the premium and $i\left(l\right)$ the indemnity paid when the loss l occurs. As we focus here on complete insurance, we also need y to be certain that is $i\left(\widetilde{l}_i\right)$ to be equal to $\widetilde{l}_i + c$, where c is a constant. Now, to avoid for usual problems of moral hazard, we also need policyholders not to have an incentive to declare a loss but when it occurs. We thus need $i\left(\widetilde{l}_i\right) \leqslant \widetilde{l}_i$, that is $c \leqslant 0$. So, $y = \overline{R} - \pi + c \leqslant \overline{R} - \pi$ and y is upward bounded by $\overline{R}$.

Moreover, in this context, the equal sharing of resources provided by the mutual agreement corresponds to full mutualization of losses as then $\dfrac{\widetilde{\omega}}{n} = \overline{R} - \dfrac{\sum_i \widetilde{l}_i}{n}$

1.2.3 The Incentive Constraint and the Profit of the Insurance Company

Let us suppose a two-stage game where

- at $t = 1$: the insurance company raises capital K and offers a contract (y, K)
- at $t = 2$: the policyholders has to choose whether to stay in the mutual or to go to the entrant insurance company .

As it insures the same risk as the pre-existing mutual agreement the entrant firm has to provide its policyholders with at least as much utility as under the equal-sharing rule among n individuals. Moreover, if the company finds it profitable to insure one agent, it is in its interest to insure the entire homogeneous population. Indeed, keeping capital per head constant, risk pooling between policyholders lowers the probability of bankruptcy. The company thus needs less capital to attract each additional policyholder.

Since the utility in the mutual firm is increasing with its number of members (see Proposition 1.1), an insurance company that manages to attract one individual can insure the entire population (we suppose here that insurance contracts are anonymous and thus that the company has to offer the same contract to each individual). To enter the market, the insurance company then only need to offer a contract (y,K) that provides - when every individuals sign it (the stock firm then fully take advantage of risk pooling among its policyholders) - at least as much utility as the mutual firm. The equal-sharing rule among n individuals then defines the lower bound of what the insurance company needs to provide to its policyholders. To enter the market it thus has to offer a contract (y,K) satisfying the following incentive constraint:

$$[1 - F(n.y - K)].u(y) + \int_{-\infty}^{n.y-K} u\left(\frac{\omega + K}{n}\right) f(\omega)\, d\omega \geq E\left(u\left(\frac{\omega}{n}\right)\right) \tag{1.3}$$

This constraint implies that, oppositely to Fagart, Fombaron and Jeleva [26] only one organizational form exists at the equilibrium even if agents cannot coordinate. Even if this incentive constraint binds, our model does not bring out coexistence of mutual with stock firms at the equilibrium, as then

- either every policyholders stay in the mutual firm,
- or one policyholder moves to the stock company making it more attractive to all the others (because of Proposition 1.1) and the insurance company is the only form to perform.

Therefore, the coexistence of both organization forms at the equilibrium in Fagart, Fombaron and Jeleva [26] is fully explained by the assumption of fixed capital stock that limits the size of the insurance companies.

Interestingly, constraint (1.3) also fits with the issue of a mutual firm that wants to demutualize. As the mutual firm is owned by its policyholders, they have to agree on the change in status. With homogeneous agents this will only be the case if the contract proposed by the company gives all the policyholders at least as much as what they had in the mutual form. Our model may therefore be used to analyze the incentive for a mutual insurance to change its organizational form and become a stock company.

Remark 1.2 *If it does not hold any capital, the insurance company is unable to sell any policy as agents are then better off in the mutual firm whatever the coverage proposed by the company.*

If $K = 0$, the company can never do better than the mutual firm when it is solvent as then $y < \frac{\omega}{n}$. The only way for it to satisfy the constraint is then to always go bankrupt, that is to set $y = \overline{R} \equiv \frac{\overline{\omega}}{n}$ (where $\overline{\omega}$ represent the upper bar of the distribution of $\widetilde{\omega}$). However, in this case its behavior exactly amounts to the one of a mutual firm as, when it goes bankrupt, the company equally shares its whole resource that then only consist in $\widetilde{\omega}$. So, without capital, an insurance company can not actually exist as its optimal choice is then to act just like a mutual firm. This moreover implies that the existence of a mutual firm by itself forces the company to hold capital. In the absence of a mutual firm, an insurance firm can still make positive profits even if it does not hold capital. In this case, the outside option is for the individual to be uninsured what can be overstepped by the insurance company even when it goes bankrupt, thanks to risk pooling effects between policyholders.

So, in order to attract policyholders, an insurance company that faces an incumbent mutual agreement has to invest in capital stock before the realization of the risk variable. Its expected profit can then be written as:

$$\Pi(y, K) = \delta. \int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K \tag{1.4}$$

As this expected profit is decreasing in both y and K, the only incentive for the company to increase y and K is to attract policyholders. The fact that the insolvency probability influence policyholders' behavior leads us to study the stock of capital as a choice variable of the insurance company.

1.3 Optimal Behavior of a Single Entrant Insurance Company Facing an Incumbent Mutual Firm

The program of the entrant insurance company consists in the maximization of $\Pi(y,K)$ under the constraint that individuals subscribe its policy, that can be rewritten as:

$$C(y,K) \equiv \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \geqslant 0 \quad (1.5)$$

As this constraint is increasing in both y and K (see Remark 1.1) when profit is decreasing with those two variables, it is satisfied with equality.

The problem thus become

$$\max_{y,K} \quad \left\{\delta. \int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K\right\} \tag{1.6}$$

$$\text{s.t.} \quad \left\{ \begin{array}{l} C(y,K) \equiv \displaystyle\int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \\ \qquad\qquad + \displaystyle\int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega = 0 \end{array} \right.$$

Proposition 1.2 *Suppose that either the support of $\widetilde{\omega}$ is unbounded or the upper bound of the support $\overline{\omega}$ satisfies*

$$\frac{1-\delta}{\delta} < \frac{\displaystyle\int_{-\infty}^{\overline{\omega}} \left(u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\overline{\omega}}{n}\right)\right) f(\omega) d\omega}{u'\left(\dfrac{\overline{\omega}}{n}\right)} \tag{1.7}$$

Then there exists a unique optimal solution for program (1.6) that yields a positive profit, fully characterized by the two following equations:

$$\Phi(y,K) \equiv \int_{-\infty}^{n.y-K} \left[\frac{u'\left(\frac{\omega+K}{n}\right) - u'(y)}{u'(y)} \right] f(\omega)\, d\omega - \frac{1-\delta}{\delta} = 0 \tag{1.8}$$

$$\begin{aligned} C(y,K) \equiv & \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right) \right] f(\omega)\, d\omega \\ & + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right) \right] f(\omega)\, d\omega = 0 \end{aligned} \tag{1.9}$$

Proof*: See Appendix*

Proposition 1.2 states that, if aggregate wealth can be infinite, an insurance company can always enter a market controlled by mutual firms. However, when aggregate wealth is bounded, an insurance company may not be able to profitably enter the market.

It may first seem odd that the condition (1.7) on the existence of the equilibrium depends on the upper bound of the aggregate wealth distribution. Let us recall, however, that the insurance company offers in our setting a complete insurance for a premium corresponding to the entire revenue. The condition (1.7) can therefore be interpreted as a condition on the profitability of insuring the considered risk. The left hand side of the inequality indeed corresponds to the risk-free interest rate r (as by definition $\delta = \frac{1}{1+r}$), that is to the profitability of the outside option for investment, whereas the right hand side is increasing with the upper bound of the aggregate wealth distribution that is with the maximal possible benefits of the insurer. If this distribution is not bounded, the insurer therefore always find profitable to insure the considered risk. However, if possible benefits are not high enough relative to the risk-free interest rate, the insurer does not find profitable to enter the market.

When an insurance company can enter the market, Proposition 1.2 moreover characterizes its optimal behavior, that consists of investing capital stock K^* and proposing a coverage y^* that satisfies the first order condition $\Phi(y, K) = 0$ and the constraint $C(y, K) = 0$. One can also show that this optimal behavior is unique as the first order condition expresses an increasing relationship between the offered premium and the stock of capital whereas the constraint defines a decreasing mapping between those two variables. The direction of those two relationships characterizing the equilibrium may be intuitively explained. The fact that the profit maximization gives rise to an increasing relationship between the coverage and the stock of capital may be explained by the effect of those two variables on the insolvency probability. As an increase in coverage increases this probability, the company has to also rise the stock of capital to restore a reasonable insolvency probability. Concerning the interactions with the incumbent mutual insurer, an increase in y increases the attractiveness of the company. Thus, everything else being equal, the company can decrease its capital stock without losing any policyholders.

Figure 1.1 illustrates the first order condition and the constraint in the plan (K, y).

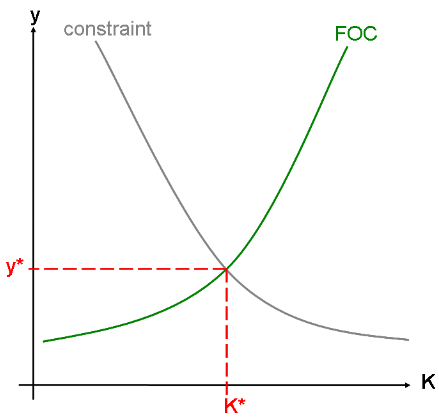


Figure 1.1: The Optimal Behavior of the Insurance Company

The reader should note that our model can easily be extended to the case of any limited liability insurance and can be used to define the amount of capital required to insure a given risk in the absence of incumbent mutual agreements. We have already seen that a limited liability insurance company optimally offers a fixed coverage when it is solvent and expends its capital stock in case of bankruptcy. Therefore, a company with capital stock k that insures a random income $\widetilde{x}$ with the fixed coverage ψ, will be insolvent when the realization of the risk is such that $x < \psi - k$. The insurer then optimally chooses the capital stock and coverage satisfying the program

$$\begin{aligned} &\max_{\psi,k} \quad \left\{ \delta . \int_{\psi-k}^{+\infty} (x+k-\psi) f(x)\, dx - k \right\} \qquad (1.10) \\ &\text{s.t.} \quad \int_{-\infty}^{\psi-k} [u(x+k) - u(x)] f(x)\, dx + \int_{y-k}^{+\infty} [u(y) - u(x)] f(x)\, dx = 0 \end{aligned}$$

The first order condition of this program that writes

$$\int_{-\infty}^{\psi-k} \left[\frac{u'(x+k) - u'(\psi)}{u'(\psi)} \right] f(x)\, dx - \frac{1-\delta}{\delta} = 0 \qquad (1.11)$$

appears to be analogous to the one of our initial problem. The following results of comparative statics (except naturally the one about n, the number of policyholders in the initial mutual agreement) can therefore be used to define the capital required by any limited liability insurance company that wants to insurance a risky activity.

1.4 Comparative Statics

In this section, we analyze the effect of different variables on the firm's optimal choices and profit. By studying how parameters of the model affect the entrant firm's profit, we are able to characterize conditions under which formal insurance companies are likely to emerge.

1.4.1 Analytical results

We first derive three analytical results, relating the insurance company's choices and profit to the cost of capital, distribution of aggregate income and degree of risk aversion.

Proposition 1.3 *:*

(i) A decrease in the cost of capital (i.e. an increase in the discount factor δ) increases the optimal capital stock (K^*) and decreases optimal proposed coverage (y^*)

(ii) Let $F_1(.)$ and $F_2(.)$ be two distributions of aggregate income. Then, if $F_1(.)$ stochastically dominates $F_2(.)$, optimal offered coverage (y^*) is higher under $F_2(.)$ than under $F_1(.)$

(iii) The profit of an entrant insurance company that wants to rule out a mutual firm is always increasing with the insured's degree of risk aversion. Moreover, when $\delta < 1$ and individuals are risk neutral, an insurance firm can not enter a market in which a mutual insurer performs.

Proof*: See Appendix.*

In providing comparative statics on δ, Proposition 1.3 first states the effect of changes in the cost of capital on the optimal choice of the insurance company: as it increases the return on invested capital (by decreasing the cost of capital), an increase in δ increases optimal capital (K^*) and then allows the company to lower y^* without increasing its insolvency probability. This is illustrated in Figure 1.2. By equation (1.7), it follows that, intuitively, it is then easier for an insurance company to emerge as δ is high, that is as capital is cheap.

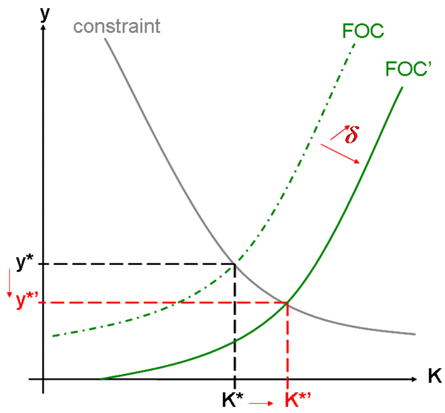


Figure 1.2: The Effect of Changes in the Discount Factor

The second part of Proposition 1.3 shows that if a distribution of aggregate income dominates (in the sense of first order stochastic dominance) another one, the optimal offered coverage increases. The effect of such a change on optimal capital stock is however ambiguous as it results from two effects:

- as it lowers the probability of low aggregate revenue and thus of bankruptcy, this change leads to a decrease in optimal invested capital
- but because of the increase in offered premia, the insurance company has to raise K^* not to increase its insolvency probability.

One can see from the proof that these effects come from the fact that the change in the distribution we study here shifts the first order condition to the North-West and the constraint to the North-Est resulting, as shown in figure 1.3, in an increase in y^* and an ambiguous effect on K^*.

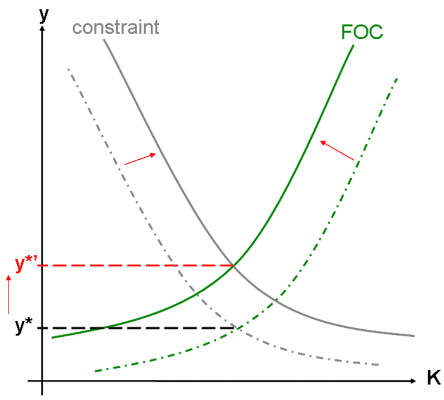


Figure 1.3: The Effect of Changes in the Distribution of Risks

As it impacts individual insurance decision, risk aversion also has an important part in the determination of the equilibrium and optimal profit. If the effect of the degree of risk aversion on choice variables is hard to grasp in the general case (see next section), Proposition 1.3 states that, as it increases expected profit, an increase in the degree of risk aversion increases the opportunity for an insurance firm to emerge. Thus, an insurance company seems to be more likely to be set up when individuals are highly risk averse. This result can be intuitively explained by the fact that an increase in individual degree of risk aversion increases the attractiveness of the insurance company that, contrary to a mutual firm, bears aggregate risk when it is solvent. Moreover, when policyholders are risk neutral, the insurance company losses then comparative advantage in bearing risk that arises from the risk neutrality of its shareholders, and can not emerge.

1.4.2 Simulations

If the effects of the variables mentioned in the previous subsection can be analytically analyzed, the effect of other variables requires the use of simulations. For exemple, the effect of the size of the population (n) is complex because it affects the distribution of aggregate income $\left(\sum_{i=1}^{n} \widetilde{\omega}_i\right)$ and we need to specify completely the relationship between the distribution of individual and aggregate income to analyze changes in n. In order to study the impact of individual risk aversion on the choice variables (Proposition 1.3 only states the effect of risk aversion on optimal profit) we also need to specify an utility function and a cumulative distribution function. For all these complex comparative statics, we use simulations with specific utility function and distribution. This also allows for the study of the impact of changes in risk aversion on the choices of insurance company.

1.4.2.1 The Retained Specification

We focus on individuals facing independent and normally distributed risks. We assume that $\widetilde{\omega}_i$ follows a $N(m, \sigma^2)$ distribution and thus that $\widetilde{\omega}$ is also distributed according to a normal distribution: $N(n.m, n.\sigma^2)$ [6]. Except in the study of changes in the variance of individual income, we analyze the case of agents' revenues with zero mean ($m = 0$) and a variance equal to one ($\sigma = 1$).

We suppose that agents have a CARA (Constant Absolute Risk Aversion) utility function:
$u(c) = -\frac{1}{\rho}.\exp(-\rho.c)$ where ρ represents the coefficient of risk aversion. When necessary, we specify a coefficient of risk aversion, ρ, equal to 0.9 [7].

As we want to study the effects of changes in different variables on the optimal behavior of the insurance company, we focus on cases where it has a high incentive to be set up, that is on situations where the cost of capital is low. We thus specify here $\delta = 0.99$.

Lastly, we have to give a specific value to n when the size of the population is not the studied variable. In those cases we specify $n = 100$ [8].

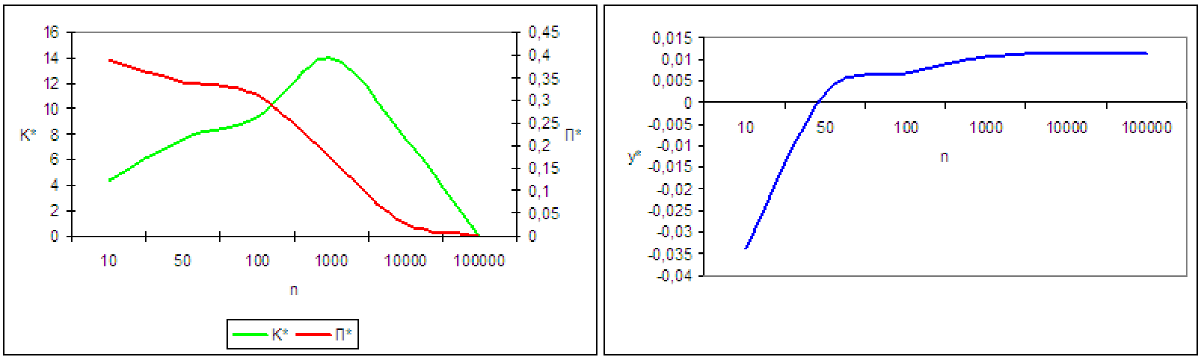
[6] The specification of normally distributed individual risks is not necessary for our simulations to be relevant. The only requirement is the aggregate risk to be distributed though a $N(n.m, n.\sigma^2)$. Given the Central Limit Theorem this can be achieved with any individual distribution for n high enough

[7] As recommended in most of recent papers (see for exemple Chetty [12] or Bombardini and Trebbi [7]) we use here a coefficient of risk aversion around one

[8] It is usually agreed to be enough for the Central Limit Theorem to hold

1.4.2.2 The Effect of Changes in the Size of the Population

As we already pointed out, the analyzis of the effect of changes in the size of the population (n) is complex because it implies changes in the distribution of aggregate income. To study the implications of such variations on the optimal behavior of the insurance company we thus need to resort to simulations (c.f. Figure 1.4 drawn on a logarithmic scale).



$(\delta = 0.99, u(c) = -\frac{1}{0.9}.\exp(-0.9.c), \widetilde{\omega} \sim N(0, n))$

Figure 1.4: The Effect of Changes in the Size of the Population

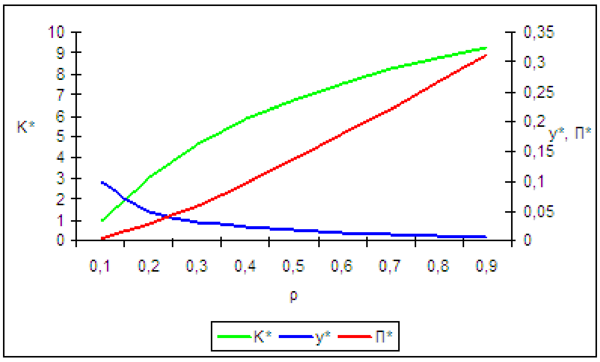
According to those simulations it appears that: *when risks are independent, an increase in the number of policyholders increases the optimal coverage offered by the insurance company and decreases its optimal profit.*

The positive effect of an increase in the number of policyholders on the offered coverage and its negative effect on optimal profit are intuitive as, by increasing risk pooling, an increase in n improves the performances of the mutual firm. The effect on the optimal stock of capital is then ambiguous as it is driven by two conflicting forces. First, as the sum of due coverage increases with n and y, the firm has to raise capital not to increase its insolvency probability. However, because it increases risk pooling, the increase in n lowers the risk of bankruptcy and the need for capital. It seems from our simulations that this last effect dominates for high values of n. Anyway, as an increase in n decreases optimal profit, it seems that it is all the more difficult for an insurance company to be set up as risk is initially shared among a lot of individuals. Therefore, as in Fagart, Fombaron and Jeleva [26] there are important network effects in our setting. However, since we endogeneize the capital stock, the number of policyholders has less impact on the expected utility in the insurance company. In Fagart, Fombaron and Jeleva [26], because of fixed capital funds, an increase in the number of policyholders has an additional negative effect, through a decrease in capital per head. Moreover, the results of these simulations give support to one of the main assumptions of Fagart, Fombaron and Jeleva [26]. They assume, with a fixed capital stock, that there exists a threshold in the number of policyholders $\overline{n}$ such that the expected utility in the company decreases with n if $n \leq \overline{n}$ and increases with n otherwise. This is confirmed by our results since the optimal capital stock is increasing in the number of policyholders when n is low, but decreasing when the company insures enough agents.

1.4.2.3 The Effect of Changes in Risk Aversion

Proposition 1.3 states the effect of changes in risk aversion on profit in the general case, but is silent on its effect on the insurance company's choice. We simulate the effect of changes in the degree of risk aversion ρ on optimal capital and coverage.

Figure 1.5 outlines the outcomes of those simulations.



$(\delta = 0.99, \widetilde{\omega} \sim N(0, n), n = 100, u(c) = -\frac{1}{\rho}.\exp(-\rho.c))$

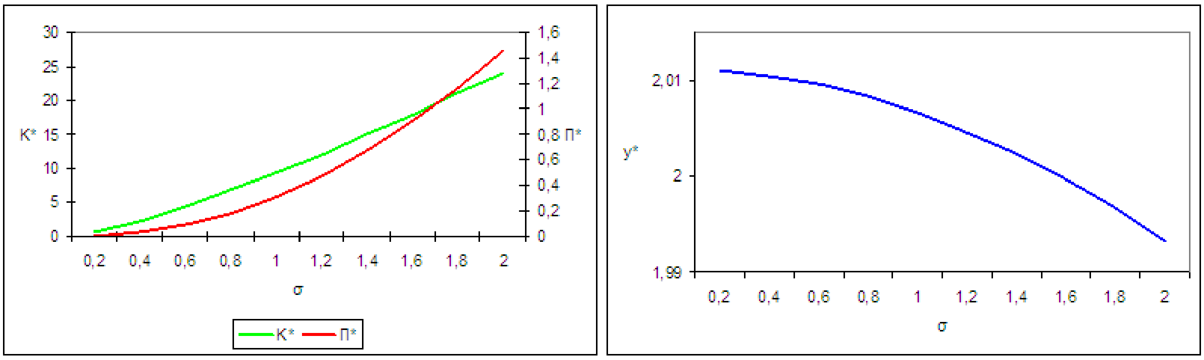
Figure 1.5: The Effect of Changes in Risk Aversion

Based on those results we learn that: *an increase in individual risk aversion increases optimal capital reserves and decreases optimal coverage.*

As already pointed out, because of the increase of their risk aversion, individuals are less demanding, and the company can propose a lower coverage. However, the result of such an increase is also to make policyholders even more reluctant to insolvency of the insurance firm that thus has to increase its capital stock. Still, as the decrease in y also has a negative effect on K, the first effect dominates. As stated in Proposition 1.3, the total influence on optimal profit is then positive. Those simulations confirm that the higher risk aversion, the larger the opportunity for an insurance firm to enter the market.

1.4.2.4 The Effect of Changes in the Variance of Individual Income

The last interesting analysis allowed by the specification of a particular cumulative distribution function concerns the effect of changes in the variance of individual income, σ.



$$(\delta = 0.99, u(c) = -\frac{1}{0.9}.\exp(-0.9.c), n = 100, \widetilde{\omega} \sim N(2.n, n.\sigma^2))$$

Figure 1.6: The Effect of Changes in the Variance of Individual Income

Those simulations give rise to the next finding: *an increase in risk (that is in the variance of each individual income distribution) increases optimal capital reserves, decreases optimal coverage and increases optimal profit.*

These interesting effects can be intuitively explained by the fact that for higher σ, the insurance company becomes more attractive with respect to the mutual firm. As it automatically raises the variance of aggregate income ($\widetilde{\omega}$) when income are independent, an increase in the variance of individual income favors the insurance company that enables policyholders to avoid aggregate risk when it is solvent. The firm can then lower offered coverage without losing policyholders. However, the effect this decrease in y has on the capital stock, seems to be offset by the need for capital induced by the increase in aggregate risk. Still, maybe because of this effect of y^* on K^*, optimal profit appears to be positively affected by this increase in risk. So, the likelihood for a company to be set up and to rule out the mutual firm is higher when risks are high.

1.5 Evidences from the Empirical Literature

To explain the co-existence of mutual with stock firms, many authors have empirically analyzed the difference between the two forms and the reasons making an entity changing from one form to another. In this section, we try to relate our results of comparative statics with the outcome of this empirical literature. Even if this is not precisely a test of our model (as we analyzed previously the entry of the first insurance company) this allows us to check if the main effects highlighted in this paper are accurate. Let us first sum up our main results of comparative statics in the following table:

Effect of...	optimal coverage	optimal capital stock	optimal profit
...a decrease in the cost of capital on	−	+	+
...an increase in individual degree of risk aversion on	−	+	+
...an increase in the number of policyholders on	+	?	−
...an increase in risk (variance of individual income) on	−	+	+

Table 1.1: Results of comparative statics

To our knowledge, no empirical study have analyzed the effect of our different parameters neither on capital stock nor on the coverage offered by insurance companies. However, it seems possible to check our results on optimal profit through the effect it has on the choice of organizational form. In our paper higher profit also means higher probability for the emergence of an insurance company. Our model thus predict that insurance companies perform in situation where capital is cheap, individual are risk adverse, risk is high and when few individual need to be insured. If information about the cost of capital and the individual degree of risk aversion are hard to grasp, several empirical papers have analyzed the effect of risk and the size of the insured population on the composition of insurance market.

Concerning the number of policyholders, the outline of our simulation seems first to be consistent with finding of Nekby [54]. Focusing on Swedish historical (1902-1910) data on health insurance, when insurance market was unregulated, this paper establishes that mutual firms were significantly larger than stock companies. This confirm our result stating that it is all the more difficult for an insurance company to emerge when risk is initially shared among a lot of policyholders. This statement is also compatible with the findings of the empirical literature on recent demutualization waves (see for example Viswanathan and Cummins, [70] or Erhemjamtsa and Leverty [24]) that asserts that an increase in competition favors demutualization. Higher competition lowering the number of policyholders in mutual firms, it may – consistently with our findings – give the mutual an incentive change its organizational form.

Our result on the effect of an increase in risk also seems to be supported by empirical researchs. Lamm-Tenant and Starks [45] for example show that, compared to mutual insurers, insurance company insures higher-risk activities (when underwriting risk is measured by the variance of loss ratio). This is moreover confirmed by Mayers and Smith [50] that analyzes 98 conversions of mutual insurers to stock forms in the US between 1920 and 1990. Using alternatively means and medians analysis and a probit regression, they assert that the probability to convert from a mutual to a stock company is higher when risk is high.

Even if those empirical works do not really form a test of a model, they corroborate some of our theoretical findings namely that insurance companies are more likely to emerge in presence of mutual agreements when risk is high and initially shared among few individuals.

1.6 Conclusion

In studying both the interaction between organizational forms in insurance and the investment choice of the stock firm, this paper highlights the relationship between insurance contracts and capital stock, through the probability of insolvency. Given this interdependence, we specify the optimal choice of coverage and capital investment of an entrant insurance company that faces an incumbent mutual agreement, and show they are unique. This paper moreover establishes that the possibility for a stock firm to rule out a mutual firm (or mutual risk-sharing arrangements) is higher as the size of the insured population and the capital cost are low, and as risk and individual risk aversion are high.

This model explains how and why some risks (or some areas) are exclusively insured through mutual or stock firms but, as it considers homogeneous agents, does not explain another feature of insurance market: the coexistence of the two organizational forms. As agents are here homogeneous, the insurance company just needs to give the same utility as the mutual firm to insure the entire population. A possible way to model the coexistence of stock and mutual insurers in our framework would be to introduce heterogeneity. Insurance companies may then attract some kinds of individuals the others staying in the mutual company. Because the agents remaining in the mutual firm are not indifferent to the emergence of an insurance company, such an extension may allow a welfare analysis that would lead to policy advices concerning the regulation of insurance markets.

Our paper also emphasizes situations where a mutual agreement is sustainable, that is when no stock company can enter the market. It moreover endogenizes the choice of capital of a stock firm when it can enter the market. However we are not able to analyze, in our setting, the entry of further companies because of difficulties in aggregating risk distributions. This issue prevents us to analyze the effect of potential shift of a policyholders from an insurance company to one other, and thus to define an equilibrium. Such a study would yet be meaningful as it would allow us to study the impact of openness to insurance companies competition of markets reserved to mutual firms. This way we would be able to study the impact of deregulations making an insurance line reserved to mutual insurers (as health insurance in France) contestable. With an exogenous capital stock, Fagart, Fombaron and Jeleva [26] already studied a similar situation. However, with an endogenous choice of capital stock, there results are likely to change mainly because of the ambiguous effect (highlighted is our paper) the number of policyholders has on optimal capital stock.

This work also seems to open interesting perspectives concerning the study of capital requirements. It appears for example worthwhile to extend our work to the study of investment insurance. Our model could then be enriched to determine the optimal amount of risky investment a company can insure with a given stock of capital. In this way it would establish the optimal investment-capital ratio, when accounting for limited liability. In the same direction, a possible extension of this work concerns the management of catastrophic risk. Our model of choice of capital under limited liability could then be useful to determine the different thresholds of the reinsurance process.

It lastly seems interesting to take into account the dynamic implications of the possibility of insolvency. In future work it would indeed be worthwhile to analyze the long term effects of bankruptcy of insurance companies on their policyholders' utility. It might be that this expected utility is no longer strictly increasing with the coverage offered by the company. The long-run effect of a bankruptcy might then make the negative influence of an increase in coverage on insolvency probability exceed the positive one it has on monetary gains in case of solvency. Moreover, the introduction of a dynamic framework might also change the optimal agreement offered by the mutual firm since, as shown by Génicot and Ray [29], it is not always optimal for mutual insurance agreements to provide equal sharing.

1.7 Appendix

1.7.1 Proof of Proposition 1.1

Let us consider individual revenues $\widetilde{\omega}_i$ with the same expectation and a combination α such that $\sum_{i=1}^{n} \alpha_i = 1$. Then, as

$$\sum_{i=1}^{n} \alpha_i.\widetilde{\omega}_i = \frac{1}{n}.\sum_{i=1}^{n} \widetilde{\omega}_i + \sum_{i=1}^{n} \left(\alpha_i - \frac{1}{n}\right) \widetilde{\omega}_i \tag{1.12}$$

we have that $\sum_{i=1}^{n} \alpha_i.\widetilde{\omega}_i$ is a mean-preserving spread of $\frac{1}{n}.\sum_{i=1}^{n} \widetilde{\omega}_i$ if:

$$E\left(\sum_{i=1}^{n} \left(\alpha_i - \frac{1}{n}\right) \widetilde{\omega}_i \Big| \sum_{k=1}^{n} \widetilde{\omega}_k\right) = 0 \tag{1.13}$$

that is if the variable $\sum_{i=1}^{n} \alpha_i.\widetilde{\omega}_i$ is equal to $\frac{1}{n}.\sum_{i=1}^{n} \widetilde{\omega}_i$ augmented by a noise with null conditional expectation.

Then, $\sum_{i=1}^{n} \alpha_i.\widetilde{\omega}_i$ is more risky than $\frac{1}{n}.\sum_{i=1}^{n} \widetilde{\omega}_i$ if:

$$\sum_{i=1}^{n} \left(\alpha_i - \frac{1}{n}\right) E\left(\widetilde{\omega}_i \Big| \sum_{k=1}^{n} \widetilde{\omega}_k\right) = 0 \tag{1.14}$$

for which a sufficient condition is:

$$E\left(\widetilde{\omega}_i \Big| \sum_{k=1}^{n} \widetilde{\omega}_k\right) = E\left(\widetilde{\omega}_j \Big| \sum_{k=1}^{n} \widetilde{\omega}_k\right) \forall i, j \tag{1.15}$$

Thus, if for each given macroscopic state $\sum_{k=1}^{n} \widetilde{\omega}_k$, individual risks have a common expectation (which is the case for independent risks), an equal sharing agreement makes everybody better off.

Moreover, equal sharing is obviously Pareto efficient since it maximizes (for instance) the utilitarian criterion:

$$\widetilde{\chi}_1^* = \widetilde{\chi}_2^* = ... = \widetilde{\chi}_n^* = \frac{\sum_{k=1}^{n} \widetilde{\omega}_k}{n} = \frac{\widetilde{\omega}}{n} = \arg\max \left\{ \frac{1}{n} \sum_{i=1}^{n} \mathrm{E}\left(u\left(\widetilde{\chi}_i\right)\right), \sum_{i=1}^{n} \widetilde{\chi}_i = \widetilde{\omega} \right\} \tag{1.16}$$

Indeed for all $\left(\widetilde{\chi}_i\right)_{i=1...n}$ such that $\sum_{i=1}^{n} \widetilde{\chi}_i = \widetilde{\omega}$, we have:

$$\begin{aligned}
\frac{1}{n} \sum_{i=1}^{n} E\left(u\left(\widetilde{\chi}_i\right)\right) &= \mathrm{E}\left(\sum_{i=1}^{n} \frac{1}{n} u\left(\widetilde{\chi}_i\right)\right) \\
&\leq \mathrm{E}\left(u\left(\frac{1}{n} \sum_{i=1}^{n} \widetilde{\chi}_i\right)\right) = \mathrm{E}\left(u\left(\frac{\widetilde{\omega}}{n}\right)\right) \\
\text{that is} &: \\
\frac{1}{n} \sum_{i=1}^{n} E\left(u\left(\widetilde{\chi}_i\right)\right) &\leq \frac{1}{n} \sum_{i=1}^{n} E\left(u\left(\frac{\widetilde{\omega}}{n}\right)\right)
\end{aligned}$$

As individuals are the owner of the mutual firm, equal sharing of resources is then optimal.

The proof of the second part of Proposition 1.1 is provided in Fagart, Fombaron and Jeleva [26].

1.7.2 Proof of Proposition 1.2

1.7.2.1 First Order Conditions

The program of the insurance company being:

$$\begin{aligned}
\max_{y,K} \quad & \Pi(y,K) \equiv \delta. \int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K \\
\text{s.t.} \quad & C(y,K) \equiv \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega + K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \\
& + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega = 0
\end{aligned} \tag{1.17}$$

its first order condition can be written as:

$$\begin{aligned}
& -\frac{\partial \Pi(y,K)/\partial K}{\partial C(y,K)/\partial K} = -\frac{\partial \Pi(y,K)/\partial y}{\partial C(y,K)/\partial y} \\
\Leftrightarrow \quad & \frac{1-\delta[1-F(n.y-K)]}{\frac{1}{n}\int_{-\infty}^{n.y-K} u'\left(\frac{\omega+K}{n}\right) f(\omega)\, d\omega} = \frac{n.\delta}{u'(y)} \\
\Leftrightarrow \quad & \frac{1-\delta[1-F(n.y-K)]}{\delta F(n.y-K)} = \frac{E\left(u'\left(\frac{\omega+K}{n}\right) | \frac{\omega+K}{n} \leq y\right)}{u'(y)} \\
\Leftrightarrow \quad & \Phi(y,K) \equiv \int_{-\infty}^{n.y-K} \left[\frac{u'\left(\frac{\omega+K}{n}\right) - u'(y)}{u'(y)}\right] f(\omega)\, d\omega - \frac{1-\delta}{\delta} = 0
\end{aligned}$$

1.7.2.2 Second Order Conditions

It can then be proved that this first order condition along with the constraint correspond to necessary and sufficient conditions that describe a maximum.

To do so let us study the following larger problem:

$$\begin{aligned}
\max_{y,K,z(.)} \quad & \int_{-\infty}^{+\infty} \omega f(\omega) d\omega - n\int_{-\infty}^{+\infty} z(\omega) f(\omega) d\omega - \frac{1-\delta}{\delta}K \\
\text{s.t.} \quad & \begin{cases} \int_{-\infty}^{+\infty} u(z(\omega)) f(\omega) d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega) d\omega \\ \min\left(\frac{\omega+K}{n}, y\right) \geq z(\omega)\ \forall\omega \end{cases}
\end{aligned} \tag{1.18}$$

The objective function of (1.18) $\left(V(K,y,z(.)) = \int_{-\infty}^{+\infty} \omega f(\omega) d\omega - n\int_{-\infty}^{+\infty} z(\omega) f(\omega) d\omega - \frac{1-\delta}{\delta}K\right)$ is linear in K, y and $z(.)$. Thus if the constraints define a convex set, then problem (1.18) is regular and its first order conditions are both necessary and sufficient for a maximum.

To verify whether it is true let us consider two triplets $K_1, y_1, z_1(.)$ et $K_2, y_2, z_2(.)$ that satisfy the constraints:

$$\left\{ \begin{array}{c} \int_{-\infty}^{+\infty} u(z_1(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega \\ \min\left(\frac{\omega + K_1}{n}, y_1\right) \geq z_1(\omega)\ \forall\omega \end{array} \right. \tag{1.19}$$

$$\left\{ \begin{array}{c} \int_{-\infty}^{+\infty} u(z_2(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega \\ \min\left(\frac{\omega + K_2}{n}, y_2\right) \geq z_2(\omega)\ \forall\omega \end{array} \right. \tag{1.20}$$

Then:

1. $\forall\alpha \in [0,1]$
   $\alpha \int_{-\infty}^{+\infty} u(z_1(\omega))f(\omega)d\omega + (1-\alpha)\int_{-\infty}^{+\infty} u(z_2(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega,$
   $\alpha \int_{-\infty}^{+\infty} u(z_1(\omega))f(\omega)d\omega + (1-\alpha)\int_{-\infty}^{+\infty} u(z_2(\omega))f(\omega)d\omega$
   $\leq \int_{-\infty}^{+\infty} u(\alpha z_1(\omega) + (1-\alpha)z_2(\omega))f(\omega)d\omega$
   and thus: $\int_{-\infty}^{+\infty} u(\alpha z_1(\omega) + (1-\alpha)z_2(\omega))f(\omega)d\omega \geq \int_{-\infty}^{+\infty} u\left(\frac{\omega}{n}\right) f(\omega)d\omega$
2. Similarly, $\forall\alpha \in [0,1]$, we have
   $\alpha \min\left(\frac{\omega + K_1}{n}, y_1\right) + (1-\alpha)\min\left(\frac{\omega + K_2}{n}, y_2\right) \geq \alpha z_1(\omega) + (1-\alpha)z_2(\omega)$
   Then, the two variables function $\min(a,b)$ being concave from $\mathbb{R}^2$ into $\mathbb{R}$, it follows that:
   $\alpha \min\left(\frac{\omega + K_1}{n}, y_1\right) + (1-\alpha)\min\left(\frac{\omega + K_2}{n}, y_2\right)$
   $\leq \min\left(\frac{\omega + \alpha K_1 + (1-\alpha)K_2}{n}, \alpha y_1 + (1-\alpha)y_2\right)$
   Which leads to: $\min\left(\frac{\omega + \alpha K_1 + (1-\alpha)K_2}{n}, \alpha y_1 + (1-\alpha)y_2\right) \geq \alpha z_1(\omega) + (1-\alpha)z_2(\omega)$

Thus, the constraints of program (1.18) define a convex set and, as already pointed out, its first order conditions are both necessary and sufficient for a maximum.

Now, to prove that the equations $\Phi(y, K) = 0$ and $C(y, K) = 0$ define the optimum choice of the insurance company we have to prove that they satisfy the first order conditions of the program (1.18). Indeed, if those equations define the maximum of program (1.18) that is larger than (1.17) in imposing $\min\left(\frac{\omega+K}{n}, y\right) \geq z(\omega)\ \forall\omega$ instead of $\min\left(\frac{\omega+K}{n}, y\right) \equiv z(\omega)\ \forall\omega$, they also define the maximum of (1.17).

Calling λ and $\mu(\omega)f(\omega)$ the multipliers associated with the constraints, the first order conditions of (1.18) can be written as:

$$\forall\omega, \qquad -nf(\omega) + \lambda u'(z(\omega)f(\omega) = \mu(\omega)f(\omega) \tag{1.21}$$

$$\int_{ny-K}^{+\infty} \mu(\omega)f(\omega)d\omega = 0 \tag{1.22}$$

$$-\frac{1-\delta}{\delta} + \frac{1}{n}\int_{-\infty}^{ny-K} \mu(\omega)f(\omega)d\omega = 0 \tag{1.23}$$

the complementarity conditions being:

$$\lambda \geq 0 \tag{1.24}$$

$$\mu(\omega)f(\omega) \geq 0 \tag{1.25}$$

$$\lambda\int_{-\infty}^{+\infty} \left(z(\omega) - u\left(\frac{\omega}{n}\right)\right) f(\omega)d\omega = 0 \tag{1.26}$$

$$\mu(\omega)f(\omega)\left(\min\left(\frac{\omega+K}{n}, y\right) - z(\omega)\right) = 0 \tag{1.27}$$

(1.21) together with $\mu(\omega) \geq 0$ then leads to $\omega \geq ny - K \Longrightarrow \mu(\omega)f(\omega) = 0$

which gives, using (1.22): $\omega \geq ny - K \Longrightarrow z(\omega) = cte = z$

We then have:

$$\begin{aligned}
\lambda u'(z) &= n && (1.28)\\
\frac{1}{n}\int_{-\infty}^{ny-K}[-n+\lambda u'(z(\omega))]f(\omega)d\omega &= \frac{1-\delta}{\delta} && (1.29)\\
\mu(\omega)f(\omega)\left(\frac{\omega+K}{n}-z(\omega)\right) &= 0 \text{ for } \omega \le ny-K && (1.30)\\
z(\omega) &= z \text{ for } \omega \ge ny-K && (1.31)\\
\lambda(\int_{-\infty}^{+\infty}\left(u(z(\omega))-u\left(\frac{\omega}{n}\right)\right)f(\omega)d\omega &= 0 && (1.32)
\end{aligned}$$

And one then can see that $\lambda, z(\omega), y, K$ verifying:

$$\begin{aligned}
\lambda u'(y) &= n && (1.33)\\
\frac{1}{n}\int_{-\infty}^{ny-K}\left[-n+\lambda u'\left(\frac{\omega+K}{n}\right)\right]f(\omega)d\omega &= \frac{1-\delta}{\delta} && (1.34)\\
z(\omega) &= \min\left(\frac{\omega+K}{n}, y\right) && (1.35)\\
\int_{-\infty}^{+\infty}\left(u(z(\omega))-u\left(\frac{\omega}{n}\right)\right)f(\omega)d\omega &= 0 && (1.36)
\end{aligned}$$

(that are the solutions of (1.17)) are solutions of previous equations (that is of (1.18)) with $y=z$.

Thus, the solutions of the program of the insurance company being the maximum of a larger program containing the one of the firm, it is also the maximum our this first program.

1.7.2.3 The Existence of An Optimum

Once the optimum characterized we need to focus on the conditions under which an optimum giving a positive expected profit exists. As the company can always make null profit by mimicking a mutual firm in setting $K = 0$ and $y = \frac{\overline{\omega}}{n}$ (as explained in section 2.2, to avoid for moral hazard, the company has to propose $y \leqslant \overline{R} \equiv \frac{\overline{\omega}}{n}$), this will be the case when there exists an optimum different from $K = 0$ and $y = \frac{\overline{\omega}}{n}$.

Rewriting the first order condition as: $\int_{-\infty}^{ny-K} \left(u'\left(\frac{\omega + K}{n}\right) - u'(y) \right) f(\omega)d\omega = \frac{1-\delta}{\delta} u'(y)$ we can see that the left hand side is increasing in y and decreasing in K, when the right hand side is decreasing in y and independent on K. Thus, a necessary condition for a solution not to exist is that for $K = 0$ and $y = \frac{\overline{\omega}}{n}$ the right hand side to be strictly lower than the left hand side, that is $\int_{-\infty}^{\overline{\omega}} \left(u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\overline{\omega}}{n}\right) \right) f(\omega)d\omega < \frac{1-\delta}{\delta} u'\left(\frac{\overline{\omega}}{n}\right)$.

So,

- if $\widetilde{\omega}$ is not bounded ($\overline{\omega} = +\infty$), as the utility function satisfies the Inada condition, $u'(+\infty) = 0$ and an optimum that gives positive profit always exists
- if $\widetilde{\omega}$ is upward bounded ($\overline{\omega} < +\infty$) equilibrium exists only if $\frac{1-\delta}{\delta} < \frac{\int_{-\infty}^{\overline{\omega}} \left(u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\overline{\omega}}{n}\right) \right) f(\omega)d\omega}{u'\left(\frac{\overline{\omega}}{n}\right)}$

1.7.2.4 Uniqueness of The Optimum

Rewriting the first order condition of the firm's program as:

$$\frac{E\left(u'\left(\frac{\widetilde{\omega} + K}{n}\right) | \frac{\widetilde{\omega} + K}{n} \leq y \right)}{u'(y)} = \frac{1 - \delta.[1 - F(n.y - K)]}{\delta.F(n.y - K)} \tag{1.37}$$

one can then show that when it holds, this condition corresponds to a unique mapping between the offered premia (y) and the capital stock (K). Indeed, keeping K constant, the left hand side is increasing in y from 1 to $+\infty$, when the right hand side is decreasing from $+\infty$ to 1. So, for each value of stock of capital the first order condition of studied programm gives a unique optimal premium.

Moreover,

- As:
$\dfrac{\partial \Phi(y,K)}{\partial K} = \dfrac{1}{n.u'(y)} \displaystyle\int_{-\infty}^{n.y-K} u'' \left(\frac{\omega + K}{n}\right) f(\omega)\, d\omega < 0$, and
$\dfrac{\partial \Phi(y,K)}{\partial y} = -\dfrac{u''(y)}{u'(y)^2} \displaystyle\int_{-\infty}^{n.y-K} u' \left(\frac{\omega + K}{n}\right) f(\omega)\, d\omega > 0$,
the first order condition of our problem gives rise to an increasing relationship between y and K $\left(\dfrac{\partial y}{\partial K} \equiv -\dfrac{\partial \Phi(y,K)/\partial K}{\partial \Phi(y,K)/\partial y} > 0\right)$.
- Likewise, as
$\dfrac{\partial C(y,K)}{\partial K} = \dfrac{1}{n}.\displaystyle\int_{-\infty}^{n.y-K} u' \left(\frac{\omega + K}{n}\right) f(\omega)\, d\omega > 0$
$\dfrac{\partial C(y,K)}{\partial y} = [1 - F(n.y - K)].u'(y) > 0$
the constraint corresponds to an decreasing relationship between y and K
$\left(\dfrac{\partial y}{\partial K} \equiv -\dfrac{\partial C(y,K)/\partial K}{\partial C(y,K)/\partial y} < 0\right)$.

So, the optimal behavior of an entrant insurance company that competes with an incumbent mutual firm, characterized by the two equations $\Phi(y,K) = 0$ and $C(y,K) = 0$, is unique (when it exists).

1.7.3 Proof of Proposition 1.3

1.7.3.1 Effect of Changes in δ

As the optimal choice of the company is fully characterized by the two equations $\Phi(y,K) = 0$ and $C(y,K) = 0$, and as $C(y,K)$ is independent on δ, the effect of the capital cost (δ) on the equilibrium has to verify:

$$\left\{ \begin{array}{l} \dfrac{\partial C(y,K)}{\partial y}.\dfrac{\partial y}{\partial \delta} + \dfrac{\partial C(y,K)}{\partial K}.\dfrac{\partial K}{\partial \delta} = 0 \\ \dfrac{\partial \Phi(y,K,\delta)}{\partial y}.\dfrac{\partial y}{\partial \delta} + \dfrac{\partial \Phi(y,K,\delta)}{\partial K}.\dfrac{\partial K}{\partial \delta} + \dfrac{\partial \Phi(y,K,\delta)}{\partial \delta} = 0 \end{array} \right. \tag{1.38}$$

That is:

$$\frac{\partial y}{\partial \delta} = \frac{\frac{\partial C(y,K)}{\partial K}.\frac{\partial \Phi(y,K,\delta)}{\partial \delta}}{\frac{\partial C(y,K)}{\partial y}.\frac{\partial \Phi(y,K,\delta)}{\partial K} - \frac{\partial C(y,K)}{\partial K}.\frac{\partial \Phi(y,K,\delta)}{\partial y}} \tag{1.39}$$

and,

$$\frac{\partial K}{\partial \delta} = \frac{-\frac{\partial C(y,K)}{\partial y}.\frac{\partial \Phi(y,K,\delta)}{\partial \delta}}{\frac{\partial C(y,K)}{\partial y}.\frac{\partial \Phi(y,K,\delta)}{\partial K} - \frac{\partial C(y,K)}{\partial K}.\frac{\partial \Phi(y,K,\delta)}{\partial y}} \tag{1.40}$$

Now as, $\frac{\partial C(y,K)}{\partial y} > 0$, $\frac{\partial C(y,K)}{\partial K} > 0$, $\frac{\partial \Phi(y,K,\delta)}{\partial y} > 0$, $\frac{\partial \Phi(y,K)}{\partial K} < 0$ and $\frac{\partial \Phi(y,K,\delta)}{\partial \delta} = \frac{1}{\delta^2} > 0$ one ends up with $\frac{\partial y}{\partial \delta} < 0$ and $\frac{\partial K}{\partial \delta} > 0$.

1.7.3.2 (Effect of Changes in The Distribution of $\widetilde{\omega}$

If $F_2(\widetilde{\omega})$ first-order stochastically dominates $F_1(\widetilde{\omega})$ $\left(F_1(\widetilde{\omega}) \geq F_2(\widetilde{\omega}) \forall \widetilde{\omega} \in]-\infty, +\infty[\right)$ then:

- The first order condition: $\Phi(y,K) = 0$ can be written as:

$$\int_{-\infty}^{n.y-K} u'\left(\frac{\omega+K}{n}\right) f(\omega)\, d\omega - \frac{1-\delta\left[1-F(n.y-K)\right]}{\delta} u'(y) = 0 \tag{1.41}$$

that is, after integrating by parts: $\Phi_p(y,K,F) \equiv -\int_{-\infty}^{ny-K} u''\left(\frac{\omega+K}{n}\right) F(\omega)\, d\omega - \frac{1-\delta}{\delta} u'(y) = 0$

Thus,

$$\Phi_p(y,K,F_1) - \Phi_p(y,K,F_2) = \int_{-\infty}^{ny-K} \underbrace{\left[F_2(\omega) - F_1(\omega)\right]}_{<0 \text{ by assumption}} \underbrace{u''\left(\frac{\omega+K}{n}\right)}_{<0} d\omega \geq 0 \tag{1.42}$$

- Similarly, integrating by parts the incentive constraint $C(y, K) = 0$, one gets:

$$\begin{aligned} C_p(y, K, F) \equiv\; & u(y) - \lim_{a \to +\infty} u(a) + \int_{-\infty}^{n.y-K} \left[u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\omega + K}{n}\right)\right] F(\omega)\, d\omega \\ & + \int_{ny-K}^{+\infty} u'\left(\frac{\omega}{n}\right) F(\omega)\, d\omega = 0 \end{aligned} \quad (1.43)$$

And,

$$\begin{aligned} C_p(y, K, F_1) - C_p(y, K, F_2) =\; & \int_{-\infty}^{n.y-K} \left[u'\left(\frac{\omega}{n}\right) - u'\left(\frac{\omega + K}{n}\right)\right] [F_1(\omega) - F_2(\omega)]\, d\omega \\ & + \int_{ny-K}^{+\infty} u'\left(\frac{\omega}{n}\right) [F_1(\omega) - F_2(\omega)]\, d\omega \geq 0 \end{aligned} \quad (1.44)$$

Using (1.42) and (1.44) one can then prove by contradiction that changing the distribution from $F_1(.)$ to $F_2(.)$ leads to a increase in the optimal offered coverage.

Indeed, if $y_1^* > y_2^*$ and F_2 first-order stochastically dominates F_1 then, according to the first order condition, we necessarily have $SP(y_1^*, K_2^*, F_1) = SP(y_2^*, K_2^*, F_2) = 0$ which means, from (1.42) and as $\frac{\partial \Phi_p(y, K, F)}{\partial y} > 0$ and $\frac{\partial \Phi_p(y, K, F)}{\partial K} > 0$, that the company will optimally choose $K_1^* < K_2^*$.

However, under the same assumption, according to the incentive constraint, we also necessarily have that $C_p(y_1^*, K_2^*, F_1) = C_p(y_2^*, K_2^*, F_2) = 0$. Now, with (1.44) and as $\frac{\partial C_p(y, K, F)}{\partial y} > 0$ and $\frac{\partial C_p(y, K, F)}{\partial K} > 0$ this would mean that $K_1^* > K_2^*$ which enters in contradiction with the previous result.

Thus, if F_2 first-order stochastically dominates F_1, then the company will necessarily choose $y_1^* < y_2^*$.

1.7.3.3 Effect of Changes in individuals degree of risk aversion

In the program of the insurance company:

$$\begin{aligned}
&\max_{y,K} \quad \left\{\delta. \int_{n.y-K}^{+\infty} (\omega + K - n.y)\, f(\omega)\, d\omega - K\right\} &(1.45)\\
&\text{s.t.} \quad C(y,K,u(.)) \equiv \int_{-\infty}^{n.y-K} \left[u\left(\frac{\omega+K}{n}\right) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega + \int_{n.y-K}^{+\infty} \left[u(y) - u\left(\frac{\omega}{n}\right)\right] f(\omega)\, d\omega \geq 0
\end{aligned}$$

the objective function being independent of insureds utility function, the effect of risk aversion on optimal profit only goes through the constraint.

Let us take a strictly increasing and concave function g and set $w = g \circ v$. We then have that w is a Von Neumann Morgenstern utility function of a more risk averse individual than v.

We now can prove that because $C(y,K,w(.)) \geq C(y,K,v(.))$, that is because it enlarge the set of possible choices, an increase in individual risk aversion increase optimal profit.

Indeed as g is increasing and concave:

$$\begin{aligned}
w\left(\frac{\omega+K}{n}\right) - w\left(\frac{\omega}{n}\right) &= g\left(v\left(\frac{\omega+K}{n}\right)\right) - g\left(v\left(\frac{\omega}{n}\right)\right)\\
&\geq g'\left(v\left(\frac{\omega+K}{n}\right)\right).\left(v\left(\frac{\omega+K}{n}\right) - v\left(\frac{\omega}{n}\right)\right)\\
&\geq g'(v(y)).\left(v\left(\frac{\omega+K}{n}\right) - v\left(\frac{\omega}{n}\right)\right) \qquad \forall \omega < n.y - K\\
\Rightarrow \int_{-\infty}^{n.y-K} \left[w\left(\frac{\omega+K}{n}\right) - w\left(\frac{\omega}{n}\right)\right] f(\omega)d\omega &\geq \int_{-\infty}^{n.y-K} \left[v\left(\frac{\omega+K}{n}\right) - v\left(\frac{\omega}{n}\right)\right] f(\omega)d\omega \qquad (1.46)\\
&\text{and}\\
w(y) - w\left(\frac{\omega}{n}\right) &= g(v(y)) - g\left(v\left(\frac{\omega}{n}\right)\right)\\
&\geq g'(u(y)).\left(v(y) - v\left(\frac{\omega}{n}\right)\right)\\
\Rightarrow \int_{n.y-K}^{+\infty} \left[w(y) - w\left(\frac{\omega}{n}\right)\right] &\geq \int_{n.y-K}^{+\infty} \left[v(y) - v\left(\frac{\omega}{n}\right)\right] \qquad (1.47)
\end{aligned}$$

which together leads to $C(y,K,w(.)) \geq C(y,K,v(.))$.

It follows that if (y, K) is acceptable for a given individual, it is also acceptable for a more risk averse one. Thus, the profit of an insurance company increases with individuals risk aversion.

The second part of the proposition is obvious. Indeed, if individuals are risk neutral, the constraint becomes

$$\int_{-\infty}^{n.y-K} \left[\frac{\omega+K}{n} - \frac{\omega}{n}\right] f(\omega)\, d\omega + \int_{n.y-K}^{+\infty} \left[y - \frac{\omega}{n}\right] f(\omega)\, d\omega \geq 0 \tag{1.48}$$

As the constraint is binding, it then leads to $\int_{n.y-K}^{+\infty} (\omega - n.y) f(\omega) d\omega = K.F(n.y-K)$ and the profit becomes $\Pi = -(1-\delta).K$

The optimal choice is thus to set $K = 0$, and $y = \frac{\overline{\omega}}{n}$, which exactly amount to the behavior of a mutual firm.

Chapter 2

Mutual insurance with asymmetric information: The case of adverse selection[1]

2.1 Introduction

Mutual risk sharing is certainly the most ancient way for people to reduce risk they individually face. For instance, solidarity funds or relief funds in the Middle Age were used by guilds to mitigate risk faced by workers. Nowadays, mutual insurance or assistance funds still rely on such agreements. Even regular insurance companies are led to group themselves in mutual agreements for reinsurance purposes. These pools of insurance companies are designed to share their individual residual risk without calling for a reinsurance company. Such mutual agreements rely on the very simple and general but very powerful mechanism of risk diversification. For instance when individual losses are identically distributed, the average loss, that is the total loss divided equally, is less risky (in the sense of second order stochastic dominance) than individual risk: it has the same mean but a more concentrated distribution. Sharing the total risk equally between members has the property to cancel idiosyncasic risk: the loss born only depends on the total loss, and not on its particular incidence among the population.

[1] This chapter reviews a joint work Dominique Henriet, registered as *GREQAM working paper* n°2008-11

When the individual losses are not identically distributed, this risk-reducing tool is hindered by an alteration of the individual expected loss. Indeed, sharing losses with someone who is more frequently hit, even if it reduces the variance, increases the expected loss. The advantage of risk reduction is mitigated, for one of the party of the agreement, by an increase in expected loss. The stability of a mutual agreement hence highly relies on the homogeneity of the insured population.

Equal sharing is obviously not the only way to share risk. The mutuality principle, (see for instance Gollier [30] or the seminal paper of Borch [9]) which gives a necessay condition for Pareto efficiency, in a perfect information context, states that the risk allocation must be such that individual wealth must only depend on aggregate wealth and not on each particular individual state.

This is, in some sense, the way mutual insurance firms manage risk: oppositely to an insurance company, a mutual organization (i) is owned by its risk-averse policyholders, redistributes profit and thus does not hold any capital and (ii) can make contracts depend on aggregate risk by using, for instance, ex-post contributions. The stability of such an organization relies on its ability to design contracts. A simple equal sharing rule, very powerful in reducing risk, can be unstable if the group is too heterogenous, because less risky members may not wish to sign the agreement as they may find more profitable alternative agreements.

What can do the mutual organization to avoid this simple adverse-selection effect? The answer to this question obviously crucially depends on the information. With "no information", that is when participants don't know their risk but have no reason to think they are different, the equal sharing rule is the good rule.[2] On the other hand, under perfect information, that is when people observe both their risk and the risk of other participants, heterogeneity can lead to the non optimality of the equal sharing rule.

[2] In this situation, a mutual organization is more efficient than an insurance company who cannot propose any contract being unable to compute actuarial premia.

What happens in the intermediate cases, that is when information on risk is private? In this chapter, we intend to investigate this question and specify the optimal contract proposed by a mutual insurer to two heterogeneous agents under incomplete information. We model the mutual organization as a principal facing two agents. In this context we are looking at the optimal Bayesian incentive sharing scheme between two agents that may differ in their risk exposure. We then focus on the impact of asymmetric information on efficiency. We notably analyze in which situation the Pareto efficient equal-sharing rule can be sustainable and analyze to what extend the mutuality principle holds.

The first result of this paper is that, even under complete information, equal sharing of resources is not achievable if heterogeneity is large and/or if risk aversion is low. Still, if risk exposure is public information, the optimal sharing rule always satisfies the mutuality principle, in the sense that ex-post allocation only depends on aggregate wealth. In this case, the mutual agreement is not symmetric but can be obtained by specifying payments that only depend on the aggregate loss and not on the individual loss.

This is no longer the case when we consider asymmetric information. The asymmetry of information has no impact on the optimal allocation when it consists in equal sharing of wealth. In this sense, mutual insurance better cope with asymmetric information than insurance companies (for which, as shown in Rothschild and Stiglitz [62] and Stiglitz [65], asymmetric information always leads to efficiency loss). However, when heterogeneity is too high or risk aversion too low, the introduction of asymmetric information rules out the mutuality principle. This gives an additional explanation to the failure of complete risk pooling highlighted in the empirical literature. Bayesian incentive constraints also make equal sharing unsustainable when both agents are low risk and induces some exchanges when agents have the same initial wealth level.

Finally the asymmetry of information induces changes in the direction of transfer in some state of nature for most utility functions of the HARA (Harmonic Absolute Risk Aversion) class. Therefore, when the asymmetry of information leads to a loss of efficiency, this loss is entirely bore by low risk agents, as in the case of insurance companies (see Rothschild and Stiglitz [62] and Chade and Schlee [11]).

In a similar two-type model, Rothschild and Stiglitz [62] have shown that competing insurance companies optimally offer full insurance at their fair premium to high risk individuals and give low risk insureds just as much utility for them to participate and for a separating equilibrium to exist (partial insurance at their fair premium). A recent work of Chade and Schlee [11] generalizes this property to the case of a monopolistic insurer and a continuum of types. Then, the highest type gets full coverage, all other less than full coverage. However as mutual firms can make contracts contingent on aggregate realizations, contrary to stock firms, we show in this paper that this loss of efficiency is not systematic in the case of mutual insurance. In this sense, our paper contributes to the literature on organizational form in insurance.

This paper also fits in the literature on informal insurance, and more precisely on the rejection of the hypothesis of complete risk-sharing in informal insurance. In his well-known empirical study of risk and insurance in a village in India, Townsend [68] finds a significant impact of household income on household consumption after having controlled by aggregated income. He thus rejects the hypothesis of complete informal insurance. Since this seminal paper, a large literature has investigated the reasons for this failure of full risk pooling schemes in mutual agreements. Most of the theoretical papers that focuses on this question, explain this limitation by limited commitment and assume identical agents (see Kimball [41], Coate and Ravallion [13], Kocherlakota [43], Ligon et al. [48], and Genicot and Ray [29], among others). We put forward, in this paper, asymmetric information on risk exposure as an alternative explanation.

In their study of dynamic mechanism design Doepke and Townsend [19] analyze the impact of asymmetric information but focused on moral hazard with hidden income and hidden action. In their work, hidden actions impact the probability distribution of income but agents are still ex-ante homogeneous and optimally follow the action recommended by the principal. Here, at the opposite, we want to focus on an ex-ante heterogeneous probability distribution. Genicot [28] introduces heterogeneity among agents but focuses on inequality in wealth and in risk exposure. The only paper that models this risk exposure heterogeneity is to our knowledge the work of Ligon and Thistle [47]. It however assumes equal sharing when studying the optimal size of a mutual firm (and ends with a separating equilibrium) whereas we specify the optimal sharing rule between heterogeneous agents.

Risk heterogeneity is to a greater extent taken into consideration in the literature on micro-credit. For example Townsend [69] studies the effect of moral hazard on project financing. As this paper focuses on adverse selection, the nearest work seems to be the one of Armendariz and Gollier [3] that models adverse selection in peer group borrowing. It specifies the optimal interest rate offered by competitive banks when agents are randomly paired. Then, as cross subsidization amongst borrowers acts as collateral, group borrowing lowers interest rate. In their paper however, the interest rate is the same for every type of individual and the bank is unable to extract information about the risk of the borrowers. We however model here a situation where a mutual insurer wants to exact information about the risk of its policyholders and provides the optimal risk sharing agreement.

Our model also appears meaningful in the study of reinsurance markets. In his seminal paper, Borch [8] models reciprocal reinsurance treaties as a two-persons cooperative agreements similar to ours. He then shows (Borsh [9]) that under complete information the optimal reinsurance scheme only depends on the total amount of claim, that is that the mutuality principle holds. Doherty [17] adds moral hazard to the discussion. He shows that financial tools can reduce the disincentive effects of reinsurance.

Our paper then complete the literature on reinsurance theory in focusing on adverse selection. It uses contract tools to build a reciprocal contract that give insurers the incentive to reveal their risk exposure.

Finally, our work also contributes to the literature on contract theory, by introducing behavior toward risk in the mechanism design. Indeed, in the informal insurance mechanisms, the transfers enter in the utility function. This leads to non quasi-linear preferences. It therefore adds technical issues to those usual in Bayesian implementation notably type-dependent outside option. This implies, in particular, that the objective function is not supermodulal under the contracts that satisfy the Bayesian incentive constraints.

The rest of this paper is structured as follows. Section 2 introduces the two-agent model of mutual insurance with two levels of risk. Section 3 discusses the benchmark case of complete information. In Section 4, we analyze the incomplete information case and characterize the optimal Bayesian incentive compatible sharing rule. Our conclusion and directions for future research are outlined in Section 5. Appendix A contains the proofs and we present in Appendix B the extension to the case of correlated risk types.

2.2 The Model

Consider two risk-adverse agents who face a risk on wealth. Wealth can be equal either to $\overline{x}$ or $\underline{x} = \overline{x} - d$ $(d > 0)$ in case of accident. Individual realizations are assumed to be independent[3] and follow a Bernoulli law with θ_i the probability that individual i $(i = 1, 2)$ has an accident. θ_i can take two possible values $\underline{\theta}$ and $\overline{\theta}$ with $0 < \underline{\theta} < \overline{\theta} < 1$ $\left(\Theta \equiv \{\underline{\theta}, \overline{\theta}\}\right)$.[4]

[3]Independence is not necessary for diversification. We suppose here independence for sake of simplicity.

[4]In the following individuals i with $\theta_i = \underline{\theta}$ are called low risk and those with $\theta_i = \overline{\theta}$ high risk

There are hence four states of nature ω : $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ that arise respectively with probabilities, $(1-\theta_1).(1-\theta_2)$, $\theta_1.(1-\theta_2)$, $(1-\theta_1).\theta_2$ and $\theta_1.\theta_2$. We denote $\pi(\theta_i,\theta_j,\omega)$ $(\theta_i,\theta_j) \in \left\{\underline{\theta},\overline{\theta}\right\}^2$ the probability that state ω occurs when individual 1 is of type i (either low or high risk) and individual 2 of type j. Let $X_i(\omega)$ (either equal to $\overline{x}$ or $\underline{x}$) be the initial wealth level of individual i in the state ω and $X(\omega) = X_1(\omega) + X_2(\omega)$ be the aggregate wealth. Risk types are assumed here to be independent [5] and we note $\overline{\mu} \equiv \mu(\overline{\theta}) \equiv \text{prob}(\theta_i = \overline{\theta})$ and $\underline{\mu} \equiv \mu(\underline{\theta}) \equiv \text{prob}(\theta_i = \underline{\theta}) = 1 - .\overline{\mu}$

Agents have a von Neumann utility function[6] $u(.)$ which is supposed to be twice differentiable and strictly concave.

The timing as follows:

- At date 1
  (a) a risk sharing scheme x is proposed.

  Definition 2.1 *A risk sharing scheme x specifies the way total wealth is shared among participants according to their type in each state of nature.*

$$x \; : \; \left| \begin{array}{c} \Theta^2 \times \Omega \rightarrow \mathbb{R}^2 \\ (\theta_1,\theta_2,\omega) \mapsto (x_1(\theta_1,\theta_2,\omega), x_2(\theta_1,\theta_2,\omega)) \end{array} \right.$$
$$\textit{with} \; : \; \forall \theta_1,\theta_2,\omega,\ x_1(\theta_1,\theta_2,\omega) + x_2(\theta_1,\theta_2,\omega) = X(\omega)$$

  (b) agents learn their type and chose whether or not to participate to the agreement. If they participate, they announce their type
- At date 2 risk is realized and contracts are enforced.

We moreover assume that contracts are anonymous ex-ante meaning that $x_1(\theta,\theta',(a,b)) = x_2(\theta',\theta,(b,a))$ $\forall a,b \in \{0,1\}$ and $\theta,\theta' \in \left\{\overline{\theta},\underline{\theta}\right\}$

[5] We discuss the case of correlated risk types in Appendix B

[6] As we want to focus on risk heterogeneity, and not on risk-aversion heterogeneity, we suppose that agents have the same utility function.

In the following, our model will be illustrated by a numerical example using a CARA (Constant Absolute Risk Aversion) utility function: $u(c) = -\exp(-\rho c)$ with $\rho = 1$, $\overline{x} = 3$, $\underline{x} = 1$, $\overline{\theta} = 0.3$, $\underline{\theta} = 0.1$ and $\overline{\mu} = \frac{2}{3}$.[7]

Let us first examine the benchmark case where information on individual risk is common knowledge.

2.3 The Complete Information Benchmark

In the case of complete information, two antagonistic forces are at work. First, the diversification principle pushes towards risk sharing. When $X_1(\omega)$ and $X_2(\omega)$ are identically distributed, $\frac{X_1(\omega)+X_2(\omega)}{2} = \frac{X(\omega)}{2}$ is less risky than $X_i(\omega)$. Sharing total wealth allows risk diversification and hence welfare improvement. However, in presence of risk heterogeneity, that is if $X_1(\omega)$ and $X_2(\omega)$ are not identically distributed, low risk individuals may not be willing to share the total burden. To be individually rational, the sharing scheme must then be distorted in favor of low risk agents. Assuming that agents can exit the risk-sharing agreement after they have learned their type, the risk sharing scheme must then fulfill "interim participation constraints" (IPC).

The problem of an utilitarian principal[8] is then:

$$\max_{x} \quad \sum_{\Theta^2} \mu(\theta_1)\mu(\theta_2) \sum_{\Omega} \pi(\theta_1,\theta_2,\omega)\left[u(x_1(\theta_1,\theta_2,\omega)) + u(x_2(\theta_1,\theta_2,\omega))\right] \tag{2.1}$$

$$\text{s.t.} \quad \begin{cases} x_1(\theta_1,\theta_2,\omega) + x_2(\theta_1,\theta_2,\omega) = X(\omega) & \forall \theta_1,\theta_2,\omega \\ \sum\limits_{\theta_2\in\Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\theta_1,\theta_2,\omega)\left[u(x_1(\theta_1,\theta_2,\omega)) - u(X_1(\omega))\right] \geq 0 & \forall \theta_1 \\ \sum\limits_{\theta_1\in\Theta} \mu(\theta_1) \sum\limits_{\Omega} \pi(\theta_1,\theta_2,\omega)\left[u(x_2(\theta_1,\theta_2,\omega)) - u(X_2(\omega))\right] \geq 0 & \forall \theta_2 \end{cases}$$

[7] For transfers to be possible when both agents suffer the damage, it is necessary to state $\underline{x} > 0$

[8] By definition, a mutual organization aims at satisfying its members. As here the principal does not know agents' types when offering the contract, it maximizes the above interim utilitarian program

The solution to this problem is summarized in the next proposition:

Proposition 2.1 *When information on individual risk is complete, the optimal risk sharing rule $x_1((\theta_1, \theta_2, \omega))$, $x_2(\theta_1, \theta_2, \omega)$:*

(i) *always satisfies the mutuality principle*

(ii) *corresponds to equal sharing of wealth in any configuration if risk aversion is high or heterogeneity in risk exposure is low, that is if:*

$$\frac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} \geq \frac{(1-\underline{\theta})\mathtt{E}(\theta)}{\underline{\theta}(1-\mathtt{E}(\theta))} \tag{2.2}$$

where $\widehat{x} \equiv \frac{\underline{x}+\overline{x}}{2}$ and $\mathtt{E}(\theta)$ is the ex-ante expected value of θ

(iii) *otherwise*

(a) *equal sharing is optimal when agents show the same risk type*

(b) *a low risk agent always gets more than average wealth when matched with a high risk*

Proof*: See Appendix.*

Without any participation constraint, the optimal utilitarian allocation would consist in the equal sharing rule: $x_1(\theta_1, \theta_2, \omega)) = x_2(\theta_1, \theta_2, \omega)) = X(\omega)/2 \ \forall \omega$. Considering participation constraints may make the optimal sharing rule differ from this first-best allocation. If high risk agents are always better off under the equal-sharing rule than under autarky, this may not be the case for less risky individuals. A high risk agent always profits from equal sharing as, whatever the type of the individual she is matched with, it gives her higher expected utility than remaining alone. Being paired with an other risky individual, she benefits from the above mentioned diversification principle; while if she faces a low risk agent, she is more likely to receive transfer as she experiences a higher probability of damage. This is not the case for a less risky agent. She benefits from equal sharing when being paired with an individual of the same risk type, but may loose when matched with a high risk agent.

When (2.2) is satisfied equal sharing is optimal even if individuals do not face the same risk. This inequality is quite simple to interpret. The left hand side (greater than 1 for risk averse agents) is an index of risk aversion whereas the right hand side (also greater than 1) is a measure of risk heterogeneity. It can be indeed written as $\left(1+\frac{\overline{\mu}}{\underline{\theta}(1-\mathrm{E}(\theta))}\left(\overline{\theta}-\underline{\theta}\right)\right)$. The right hand of (2.2) side therefore depends positively on the probability of being a low risk and increases with a mean-preserving spread of risk exposures $\left(\overline{\theta}-\underline{\theta}\right)$.

Equal sharing is thus optimal when heterogeneity is sufficiently low or when risk aversion is sufficiently high.

When the above mentioned inequality does not hold, that is when heterogeneity is too large, equal sharing is not individually rational for less risky agents. To be participation proof for less risky agents, the optimal risk sharing rule must give more than average wealth to the low risk individual in every state of nature (even when she suffers the damage and the other does not) when the agreement concerns two heterogenous agents $\left(x_1(\underline{\theta},\overline{\theta},\omega)\geq x_2(\underline{\theta},\overline{\theta},\omega)\forall\omega\right)$. The optimal risk sharing rule however specifies full risk sharing when agents are identical $\left(x_1(\underline{\theta},\underline{\theta},\omega)=x_1(\overline{\theta},\overline{\theta},\omega)=x_2(\overline{\theta},\overline{\theta},\omega)=x_2(\underline{\theta},\underline{\theta},\omega)=X(\omega)/2\right)$. Therefore, relative to the first best allocation (equal sharing of wealth), the introduction of participation constraints benefits to low risk agents.

Noteworthy, even when it rules out equal sharing, the optimal allocation under complete information always satisfies the mutuality principle. Indeed, the optimal allocation makes ex-post wealth only depends on aggregate realization in every configuration $\left(x_i((\theta_1,\theta_2,(a,b)))=x_i((\theta_1,\theta_2,(b,a)))\ \forall\theta_1,\theta_2\in\{\underline{\theta},\overline{\theta}\},\ a,b\in\{0,1\}\right)$. Thus, although a mutual agreement does not cancel (macroscopic) risk (oppositely to an insurance company), in this complete information setting, it fully insures individual (microscopic) risk.

These properties are confirmed by simulations on the numerical example presented above, whose results are summarized in the following table:

			agent 1			
			$\underline{\theta}$		$\overline{\theta}$	
			0	1	0	1
agent 2	$\underline{\theta}$	0	$x_1 = 3 = \overline{x}$ $x_2 = 3 = \overline{x}$	$x_1 = 2 = \widehat{x}$ $x_2 = 2 = \widehat{x}$	$x_1 \simeq 2.993 < \overline{x}$ $x_2 \simeq 3.007 > \overline{x}$	$x_1 \simeq 1.995 < \widehat{x}$ $x_2 \simeq 2.005 > \widehat{x}$
		1	$x_1 = 2 = \widehat{x}$ $x_2 = 2 = \widehat{x}$	$x_1 = 1 = \underline{x}$ $x_2 = 1 = \underline{x}$	$x_1 \simeq 1.995 < \widehat{x}$ $x_2 \simeq 2.005 > \widehat{x}$	$x_1 \simeq 0.998 < \underline{x}$ $x_2 \simeq 1.002 > \underline{x}$
	$\overline{\theta}$	0	$x_1 \simeq 3.007 > \overline{x}$ $x_2 \simeq 2.993 < \overline{x}$	$x_1 \simeq 2.005 > \widehat{x}$ $x_2 \simeq 1.995 < \widehat{x}$	$x_1 = 3 = \overline{x}$ $x_2 = 3 = \overline{x}$	$x_1 = 2 = \widehat{x}$ $x_2 = 2 = \widehat{x}$
		1	$x_1 \simeq 2.005 > \widehat{x}$ $x_2 \simeq 1.995 < \widehat{x}$	$x_1 \simeq 1.002 > \underline{x}$ $x_2 \simeq 0.998 < \underline{x}$	$x_1 = 2 = \widehat{x}$ $x_1 = 2 = \widehat{x}$	$x_1 = 1 = \underline{x}$ $x_1 = 1 = \underline{x}$

Table 2.1: The optimal agreement under complete information: a numerical example

Let us first note that condition (2.2) is violated for our numerical example as under this specification, $\frac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})} = e < \frac{(1-\underline{\theta})\mathrm{E}(\theta)}{\underline{\theta}(1-\mathrm{E}(\theta))} \simeq 2.7$. Consistently with Proposition 2.1, the optimal sharing rule therefore does not correspond to equal sharing in every state. This numerical example then illustrates the properties of the optimal agreement when equal sharing is not achievable. Indeed, the top-left and bottom-right parts of table 2.1 confirm that equal sharing is achievable when agents show the same risk type. Moreover, the top-right and bottom-left parts illustrate that (i) a low risk agent always gets more than average wealth when matched with a high risk and (ii) the mutuality principe holds as allocation are identical in configurations $(\theta_1, \theta_2, (0,1))$ and $(\theta_1, \theta_2, (1,0))$.

Before introducing asymmetric information, it seems worthwhile to note that similar results holds in the case of risk exposures (θ_1, θ_2) known before the design of the contract. Using a similar methodology, we can show that equal sharing is then optimal when $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})} \geq \dfrac{(1-\underline{\theta})\overline{\theta}}{\underline{\theta}(1-\overline{\theta})}$ (note here that the problem only arises when agents are of different types). If this inequality fails to hold, the optimal agreement still satisfies the mutuality principle and provides the low risk agent with more than average wealth $\left(x_1(\underline{\theta}, \overline{\theta}, \omega) \geq x_2(\underline{\theta}, \overline{\theta}, \omega) \forall \omega\right)$ as in the Bayesian case.

2.4 Asymmetric Information

Now turn to the incomplete information setting. When risk is private information, the risk sharing scheme must be interpreted as a mechanism. The principal has to offer a menu of contracts depending on types that gives agents the incentive to truthfully report their risk type. In our setting, a Bayesian-Nash truthfully report is a best response if :

$$\sum_{\theta_2\in\Theta}\mu(\theta_2)\sum_{\Omega}\pi(\overline{\theta},\theta_2,\omega)\left(u(x_1(\overline{\theta},\theta_2,\omega))-u(x_1(\underline{\theta},\theta_2,\omega))\right)\geq 0 \tag{2.3}$$

$$\sum_{\theta_2\in\Theta}\mu(\theta_2)\sum_{\Omega}\pi(\underline{\theta},\theta_2,\omega)\left(u(x_1(\underline{\theta},\theta_2,\omega))-u(x_1(\overline{\theta},\theta_2,\omega))\right)\geq 0 \tag{2.4}$$

$$\sum_{\theta_1\in\Theta}\mu(\theta_1)\sum_{\Omega}\pi(\theta_1,\overline{\theta},\omega)\left(u(x_2(\theta_1,\overline{\theta},\omega))-u(x_2(\theta_1,\underline{\theta},\omega))\right)\geq 0 \tag{2.5}$$

$$\sum_{\theta_1\in\Theta}\mu(\theta_1)\sum_{\Omega}\pi(\theta_1,\underline{\theta},\omega)\left(u(x_2(\theta_1,\underline{\theta},\omega))-u(x_2(\theta_1,\overline{\theta},\omega))\right)\geq 0 \tag{2.6}$$

As it only depends on realizations (and not on type), the equal sharing rule obviously satisfies these Bayesian incentive constraints. It moreover has been shown in previous section that this rule also satisfies participation constraints when heterogeneity is not too high. Thus, the next proposition is straightforward.

Proposition 2.2 *When risk is private information, equal-sharing rule is optimal when* $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\geq\dfrac{(1-\underline{\theta})\mathtt{E}(\theta)}{\underline{\theta}(1-\mathtt{E}(\theta))}$.

Therefore, in this configuration of risk, the asymmetry of information has no impact on the optimal sharing rule, and the first-best allocation is achievable when including participation and Bayesian incentive constraints. In such cases, there is no loss of efficiency due to asymmetric information. In this sense, mutual insurance better cope with asymmetric information than the case with an insurance company where asymmetric information always leads to an efficiency loss borne by low risk type (Rothschild and Stiglitz [62]).

2.4.1 Numerical Exemple

To grasp the influence of asymmetric information when equal sharing is not optimal, let us first examine our numerical example. When Bayesian incentive constraints are accounted for, the optimal sharing rule in the considered particular case can be summarized as follows:

			agent 1			
			$\underline{\theta}$		$\overline{\theta}$	
			0	1	0	1
agent 2	$\underline{\theta}$	0	$x_1 = 3 = \overline{x}$ $x_2 = 3 = \overline{x}$	$x_1 \simeq 1.985 < \widehat{x}$ $x_2 \simeq 2.015 > \widehat{x}$	$x_1 \simeq 2.988 < \overline{x}$ $x_2 \simeq 3.012 > \overline{x}$	$x_1 \simeq 1.992 < \widehat{x}$ $x_2 \simeq 2.008 > \widehat{x}$
		1	$x_1 \simeq 2.015 > \widehat{x}$ $x_2 \simeq 1.985 < \widehat{x}$	$x_1 = 1 = \underline{x}$ $x_2 = 1 = \underline{x}$	$x_1 \simeq 2.007 > \widehat{x}$ $x_2 \simeq 1.993 < \widehat{x}$	$x_1 \simeq 1.003 > \underline{x}$ $x_2 \simeq 0.997 < \underline{x}$
	$\overline{\theta}$	0	$x_1 \simeq 3.012 > \overline{x}$ $x_2 \simeq 2.988 < \overline{x}$	$x_1 \simeq 1.993 < \widehat{x}$ $x_2 \simeq 2.007 > \widehat{x}$	$x_1 = 3 = \overline{x}$ $x_2 = 3 = \overline{x}$	$x_1 = 2 = \widehat{x}$ $x_2 = 2 = \widehat{x}$
		1	$x_1 \simeq 2.008 > \widehat{x}$ $x_2 \simeq 1.992 < \widehat{x}$	$x_1 \simeq 0.997 < \underline{x}$ $x_2 \simeq 1.003 > \underline{x}$	$x_1 = 2 = \widehat{x}$ $x_1 = 2 = \widehat{x}$	$x_1 = 1 = \underline{x}$ $x_1 = 1 = \underline{x}$

Table 2.2: The optimal agreement under asymmetric information: a numerical example

It appears that asymmetric information has three main implications in the example.

First, when analyzing the top-left part of table 2.2, it comes out that equal sharing of wealth is no more optimal when both agents announce to be low risk. The optimal incentive constraint then favors, relative to the complete information benchmark, the agent that did not experience the damage.

As the allocation of wealth then depends on which agent suffers the loss (that is on individual realization), a second implication of asymmetric information is here the failure of the mutuality principle. This is moreover confirmed by the optimal sharing rule when agents announce different type as then, contrarily to the complete information case, the configuration $(\theta_1, \theta_2, (0,1))$ and $(\theta_1, \theta_2, (1,0))$ are not identical.

More surprisingly, it appears from table 2.2 that, in some configurations, the relative position of allocations with respect to average wealth changes when we introduce asymmetric information. Indeed, whereas in the complete information benchmark a low risk agent always had more than average wealth when matched with a high risk, this is not the case here. For example, in configurations $(\overline{\theta}, \underline{\theta}, (0,1))$ and $(\overline{\theta}, \underline{\theta}, (1,1))$, the high risk agent gets more than average wealth. As both agents have the same initial wealth, it even means in this last configuration that asymmetric information leads to a change in the direction of transfer.

It therefore appears stimulating to analyze to what extend these observations are generalizable. The rest of the paper is thus devoted to the analytical study of asymmetric information with a particular attention to the three previous effects.

2.4.2 The violation of the mutuality principle

In the general case, we have shown in section 2.3 that when the probability of (the opponent) being low risk is low and heterogeneity large (that is when condition (2.2) fails), the optimal sharing rule under complete information specifies $x_1(\underline{\theta}, \overline{\theta}, \omega) \geq X(\omega)/2 \geq x_1(\overline{\theta}, \underline{\theta}, \omega)$ and $x_1(\overline{\theta}, \overline{\theta}, \omega) = x_1(\underline{\theta}, \underline{\theta}, \omega) = X(\omega)/2$ $\forall \omega$. This gives high risk individuals a high incentive to cheat on their type, as they are then better off announcing $\underline{\theta}$ whatever the type announced by their opponent $\big(x_1(\underline{\theta}, \overline{\theta}, \omega) \geq x_1(\overline{\theta}, \overline{\theta}, \omega)$ and $x_1(\underline{\theta}, \underline{\theta}, \omega) \geq x_1(\overline{\theta}, \underline{\theta}, \omega)\big)$. Then, the optimal allocation under complete information does not satisfy high risk individuals Bayesian incentive constraints (2.3) and (2.5).

Assuming ex-ante anonymity, the program then becomes:

$$\max_x \quad \sum_{\Theta^2} \mu(\theta_1)\mu(\theta_2) \sum_{\Omega} \pi(\theta_1, \theta_2, \omega) \left[u(x_1(\theta_1, \theta_2, \omega)) + u(x_2(\theta_1, \theta_2, \omega))\right] \tag{2.7}$$

$$\text{s.t.} \quad \begin{cases} x_1(\theta_1, \theta_2, \omega) + x_2(\theta_1, \theta_2, \omega) = X(\omega) \qquad \forall \theta_1, \theta_2, \omega \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\overline{\theta}, \theta_2, \omega) \left[u(x_1(\overline{\theta}, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\overline{\theta}, \theta_2, \omega) \left(u(x_1(\overline{\theta}, \theta_2, \omega)) - u(x_1(\underline{\theta}, \theta_2, \omega))\right) \geq 0 \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\underline{\theta}, \theta_2, \omega) \left[u(x_1(\underline{\theta}, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\underline{\theta}, \theta_2, \omega) \left(u(x_1(\underline{\theta}, \theta_2, \omega)) - u(x_1(\overline{\theta}, \theta_2, \omega))\right) \geq 0 \end{cases} \tag{2.8}$$

The form of the solution is described in the following proposition.

Proposition 2.3 *When risk types are private information, and heterogeneity is too large* $\left(\dfrac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} < \dfrac{(1-\underline{\theta})E(\theta)}{\underline{\theta}(1-E(\theta))}\right)$

(i) The mutuality principle is not sustainable, meaning that the agents bear residual individual risk in some configurations;

(ii) The optimal agreement implies some exchanges even when agents have the same initial wealth level.

Proof*: See Appendix.*

Even when equal sharing is not achievable, we show first that there always exists a unique optimal sharing rule that improves the expected utility of both agents and gives them an incentive to truthfully report their risk type. Indeed, changing variables (by setting $h_i(\theta_1, \theta_2, \omega) \equiv u(x_i(\theta_1, \theta_2, \omega))$) allows us to show that the program of the principal admits a unique solution for $(h_1(.), h_2(.))$ and thus also for x. As the optimal allocation under complete information is not incentive compatible when (2.2) is not satisfied, one incentive compatible constraint necessarily binds.

The transfers induced at the optimum hence necessarily differ from the ones under complete information. By specifying equal sharing when both agents announce the same risk type $x_i(\theta, \theta, \omega) = \frac{X(\omega)}{2}$ and by giving more than half of the aggregate wealth to the less risky agent when individuals are of different type $x_1(\overline{\theta}, \underline{\theta}, \omega) \leq \frac{X(\omega)}{2}$, the complete information rule violates the Bayesian incentive constraint of high risk agent. To prevent her from cheating on her type, the principal *has to distort equal sharing when both agents declare to be low risk.* By giving less to the agent that suffers the damage in these cases $(x_1(\underline{\theta}, \underline{\theta}, (0,1)) > \widehat{x} > x_1(\underline{\theta}, \underline{\theta}, (1,0)))$, the contract makes less profitable for high risk individuals to announce $\underline{\theta}$. Since the optimal allocation then depends on individual realizations, this incentive scheme is done at the cost of the mutuality principle.

This result is consistent with previous empirical studies on mutual insurance in developing countries (starting with the seminal paper of Townsend [68]) that finds a significant impact of household income on household consumption after having corrected by aggregate consumption. This failure of the complete market hypothesis has mainly be explained in the literature (Coate and Ravallion [13], Kocherlakota [43], Ligon et al. [48], Dubois et al. [20]) by limited commitment and self-enforceability. When contracts can not be enforced legally (as it is the case in informal insurance in developing countries) an agent with good realizations has a high incentive to defect. She will however honor the agreement if the benefits she gets from defection are outweighed by the cost of renunciation: the breakdown of future profitable agreements. Previous works on limited commitment have shown that this is necessarily done at the cost of mutuality principle. Proposition 2.3 states that this failure can also be explained by risk heterogeneity and asymmetric information.

Here we show that that equal sharing cannot be implemented, even when both individuals are low risk. When negative risk heterogeneity is too strong, the asymmetry of information therefore induces a loss of efficiency by reducing insurance when both agents are of low type. In this sense, asymmetric information has in these configuration the same impact on mutual insurance than on insurance companies (see Rothschild and Stiglitz [62]): a reduction in the coverage offered to low risk agents.

This incomplete insurance effect has to be compensated by other transfers favorable to low risk to induce her participation to the agreement. This is done, among other things, by specifying *transfers from high risk to low risk individuals when none of the two suffer a damage*. Therefore, the optimal agreement implies some exchanges when agents have the same initial wealth level.

From a technical point of view, whatever the utility function, it first can be proven (all proofs are deferred to the appendix) that the participation constraint of low risk and the incentive constraint of high risk individuals necessarily bind at the optimum, whereas the participation constraint of high risk agents never binds. However, we can not use here the usual tools to show that only one incentive constraint binds to derive the general solution of the program. As agents' utility (through transfers) depends on both the opponent type and the realization, and because preferences are not quasi-linear, the traditional single crossing property does not hold. To overcome this difficulties, we are going to assume a particular form of the utility function. The next section will be devoted to the case of HARA preferences (see Gollier [30] and Mayer and Mayer [53]).

2.4.3 The Case of HARA Preferences

To describe more precisely the optimal agreement under asymmetric information, we need to establish which constraints bind at the optimum. To do so let us assume that agents preferences exhibit HARA, that is $u(c) = \xi\left(\eta + \frac{c}{\gamma}\right)^{1-\gamma}$ (note that $u(c)$ is defined $\forall c$ such that $\eta + \frac{z}{\gamma} > 0$, increasing and concave for $\xi(1-\gamma)\gamma^{-1} > 0$). This broad class of utility functions, labeled Hyperbolic Absolute Risk Aversion by Merton [52]) and Harmonic Absolute Risk Aversion by Gollier [30], contains Constant Relative Risk Aversion ($\eta = 0$), Constant Absolute Risk Aversion ($\gamma \to +\infty$) and logarithmic ($\gamma \to 1$) preferences as special cases.

Proposition 2.4 *Suppose that agents' preferences exhibit HARA with $\gamma \geq \frac{1}{2}$. Then, when equal sharing is not optimal, that is when (2.2) is not satisfied, the optimal sharing rule under asymmetric information is fully given by:*

$$\left\{\begin{array}{rcl} x_1(\overline{\theta},\overline{\theta},\omega) & = & X(\omega)/2 \quad \forall\omega \\ x_1(\underline{\theta},\underline{\theta},(0,1)) & > & \widehat{x} \\ \nu_1 \equiv \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} & = & \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))} < 1 \\ \nu_2 \equiv \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} & = & \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))} > 1 \end{array}\right. \tag{2.9}$$

such that the third and forth conditions of 2.8 are satisfied with equality.

This implies $\forall a \in \{0,1\}, \quad \theta \in \{\underline{\theta},\overline{\theta}\}$

$$\left\{\begin{array}{rcl} x_1(\theta,\underline{\theta},(a,0)) & \leq & x_2(\theta,\underline{\theta},(a,0)) \text{ with a strict inequality if } (a,\theta) \neq (0,\underline{\theta}) \\ x_1(\theta,\underline{\theta},(a,1)) & \geq & x_2(\theta,\underline{\theta},(a,1)) \text{ with a strict inequality if } (a,\theta) \neq (1,\underline{\theta}) \\ x_1(\overline{\theta},\overline{\theta},\omega) & = & x_2(\overline{\theta},\overline{\theta},\omega) = \frac{X(\omega)}{2} \end{array}\right.$$

Therefore, relative to the complete information benchmark, the asymmetry of information leads to a change in the direction of transfer in the state $(1,1)$ *when agents announce different types* $\left(x_1(\overline{\theta},\underline{\theta},(1,1)) > \underline{x}\right)$. Proof*: See Appendix.*

Let us examine the main results of the Proposition 2.4.

Under the HARA specification, Proposition 2.4 first states that equal sharing is optimal when both agents announce to be high risk $\left(x_1(\overline{\theta},\overline{\theta},\omega) = X(\omega)/2\ \forall\omega\right)$. This finding may be put in perspective with the one of Rothschild and Stiglitz [62] on insurance companies. Oppositely to low risk agents, risky individuals obtain their first best contract even in presence of asymmetric information. However, as mutual contracts depends on aggregate realization, this is the case in our setting only if both agents are of high risk.

When both agents are low risk, the ex-ante anonymity assumption implies that no transfer takes place when realizations are the same $(x_1(\underline{\theta},\underline{\theta},(0,0)) = \overline{x}, x_1(\underline{\theta},\underline{\theta},(1,1)) = \underline{x})$. However, as shown in Proposition 2.3, equal sharing has to be distorted when two agents that announces to be low risk experienced different realizations $(x_1(\underline{\theta},\underline{\theta},(0,1)) > \widehat{x} > x_1(\underline{\theta},\underline{\theta},(1,0)))$.

The second part of Proposition 2.4 states that this mechanism is not sufficient to prevent high risk from cheating. In the complete information setting, it has been shown that a low risk agent optimally gets more than equal sharing in any state of nature when being matched with a high risk $\left(x_1(\underline{\theta},\overline{\theta},\omega) \geq X(\omega)/2\ \forall\omega\right)$. In spite of previous distortion this still gives high risk individual an incentive to cheat. Previous mechanisms on $x_i(\underline{\theta},\underline{\theta},\omega)$ indeed reduces the incentive of cheating when a high risk individual faces a less risky agent. However, the optimal allocation under complete information also gives an incentive to cheat in the case of two risky agents as it specifies $x_1\left(\underline{\theta},\overline{\theta},\omega\right) \geq \frac{X(\omega)}{2} = x_1\left(\overline{\theta},\overline{\theta},\omega\right)$. To be incentive compatible, the optimal contract has then to provide high risk agents with more than half of the aggregate wealth in some states when agents announce different risks. To induce the participation of low risk agents, this has to be done in states relatively less likely for them, that is when the less risky agent suffers the damage: $(\overline{\theta},\underline{\theta},(0,1))$ and $(\overline{\theta},\underline{\theta},(1,1))$. Moreover, a low risk individual would still accept the agreement as the contract will still be welfare improving if she faces another low risk agent.

One interesting implication of this result is that asymmetric information then entails, in state (1,1), a change in the direction of transfer (relative to the complete information benchmark). Whereas in this state, the transfer of wealth goes from the high risk type to the low risk agent when risk types are common knowledge, the optimal agreement under asymmetric information specifies a transfer from the less to the more risky individual. For each agent, facing a declared low risk does not necessarily mean bad news. The share of the total loss will not always be in the favor of the low risk part. .

To sum up, the effect of the asymmetry of information on efficiency highly depends on the degree of heterogeneity in risk exposure. When the difference between the probabilities of damage of risk type is low, there is no loss of efficiency due to the asymmetry of information. In this sense, mutual agreements seem to be better adapted to asymmetric information than insurance companies. However, when heterogeneity is strong, the asymmetry of information leads to a loss of efficiency. As in Rothschild and Stiglitz [62] this loss is entirely born by low risk agents. When asymmetry of information causes a loss of efficiency it is hard to compare the two organizational forms. Because of the main differences between mutual and stock forms, the optimal contract described in this paper depends on the type and realization of both agents when the allocations in Rothschild and Stiglitz [62] only depend on the risk type of the involved agent.

2.5 Conclusion

Our paper contributes to both the literature on mutual insurance and mechanism design by characterizing the optimal mutual risk sharing agreement between two heterogenous agents with asymmetric information.

First, by analyzing the behavior toward risk in contracts, this work introduces the case of non quasi-linear preferences in Bayesian implementation. In spite of the technical issues it implies (mainly the non supermodularity of the objective function), we are able to solve the problem for a broad class of HARA utility functions.

Then, our paper provides an additional explanation to the failure of complete risk pooling in informal agreements. We first show that the equal sharing rule is not sustainable when risk heterogeneity is large and risk aversion is low. Still, under complete information, as ex-post wealth only depends on aggregate realization and the mutuality principle holds. However, under asymmetric information optimal mutual agreements do not prevent agents from bearing residual individual risk. Therefore, the failure of complete market observed in informal insurance may be explained by risk heterogeneity.

Another striking result of this work is that, to give agents the incentive to reveal their risk, a mutual agreement has to specify transfers in some states where agents have the same initial wealth. Finally it is has been proven that the asymmetry of information induces changes in the direction of transfer in some state of nature (relatively to the complete information benchmark) for most utility functions of the HARA class.

By analyzing the effect of asymmetric information on the efficiency of mutual agreement, this work also participates to the literature on the difference in organizational form in insurance. We show that the mutual form better copes with asymmetric information as the asymmetry of information does not necessarily implies here a loss of efficiency.

This is consistent with previous findings stating that insurance companies always perform better under complete information, but can not exist when there is no information contrary to mutual insurance. Moreover, when the asymmetry of information leads to a loss of efficiency, the loss is entirely borne by low risk type agents, as in the case of insurance companies (Rothschild and Stiglitz [62]).

In addition to these positive results, our work presents some normative implications in the design of risk-sharing contracts between financial institution or insurance companies. In particular, it can be used to precisely design the direction of conditional financial cash flows.

Part of our work seems to be generalizable to the cases of more than two agents and/or more than two realizations. First, the condition on the sustainability of equal sharing should be easily extendable to a continuum of agents or realizations. Then, equal sharing would be optimal, even under imperfect information, if it provides the less risky individual with a higher expected utility then under autarky. In this case, the less risky agent would compare his risk exposure with the average risk in the mutual agreement. When this first best is not achievable, the failure of the mutuality principle also seems generalizable. To prevent risky individual to cheat on their type it appears necessary to lower insurance for low risk agents. This seems to be necessarily done at the cost of complete risk pooling. Our model can also be extended to correlated risk exposure. An attempt to study such cases is presented in the working paper version of present work. Most of our findings (mainly propositions 1 to 3) also holds in the case of correlated types, but more attention need to be paid to the issue of single-crossing of the incentive constraints.

Is left for future research to test our findings on real data on informal insurance. It seems especially interesting to examine to what extent the mutuality principle holds in informal insurance networks. This can be done for example by studying on panel data how much the ex-post revenue of an individual depends on its initial wealth controlling for aggregate revenue. Using such a methodology, Townsend [68] finds a significant impact of household revenue on household income. This is consistent with our finding. Whether this is due risk heterogeneity or to limited commitment (as argued by Coate and Ravaillon [13], Kocherlokota [43] and Ligon et al. [48] for example) remains an open issue. One way to test it would be to analyze if some transfers take place when agents experienced the same realization. If it does, this will advocate for risk heterogeneity as shown in Proposition 2.1. Moreover, if the direction of transfer change whether both agents experience good or bad realizations, according to Proposition 2.4, this will argue for the asymmetric information explanation.

Our work also seems to have implications on micro-credit. It will therefore be interesting to extend it to a situation where a bank tries, by setting the interest rate, to extract information about whether each of two borrowers involved in a micro-credit agreement invests in a safe or a risky project.

2.6 Appendix A: The Proofs

2.6.1 Proof of Proposition 2.1

If the Interim Participation Constraints do not bind at the optimum, the solution of the utilitarian program is obviously: $x_1(\theta_1,\theta_2,\omega) = x_2(\theta_1,\theta_2,\omega) = X(\omega)/2$.

This solution satisfies Interim IPC if and only if $\forall\theta_1$:

$$\sum_{\theta_2\in\Theta}\mu(\theta_2)\left[(1-\theta_1)\theta_2\left(u(\widehat{x})-u(\overline{x})\right)+(1-\theta_2)\theta_1\left(u(\widehat{x})-u(\underline{x})\right)\right]\geq 0$$

that is if $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\geq\dfrac{(1-\theta_1)E(\theta_2)}{\theta_1(1-E(\theta_2))}$

Now, as $\widehat{\theta}\leq\widetilde{\theta}$, $\dfrac{(1-\overline{\theta})E(\theta)}{\overline{\theta}(1-E(\theta))}\leq 1\leq\dfrac{(1-\underline{\theta})E(\theta)}{\underline{\theta}(1-E(\theta))}$ and the IPC is always verified for $\theta_1=\overline{\theta}$. $\left(\frac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\geq 1\right)$. For $\theta_1=\underline{\theta}$, the equal sharing rule satisfies the IPC if: $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\geq\dfrac{(1-\underline{\theta})E(\theta)}{\underline{\theta}(1-E(\theta))}$

When this inequality does not hold, that is if the probability of being high risk $\overline{\mu}$ and the heterogeneity in the risk exposure are high $\left(\dfrac{(1-\underline{\theta})E(\theta)}{\underline{\theta}(1-E(\theta))}=1+\dfrac{\overline{\mu}(\overline{\theta}-\underline{\theta})}{\underline{\theta}(1-E(\theta))}\right.$ increases with a mean-preserving spread of $\left.(\overline{\theta}-\underline{\theta})\right)$, and risk aversion is low, equal sharing is not individually rational and the program becomes:

$$\max_x\quad\sum_{\Theta^2}\mu(\theta_1)\mu(\theta_2)\sum_{\Omega}\pi(\theta_1,\theta_2,\omega)\left[u(x_1(\theta_1,\theta_2,\omega))+u(x_2(\theta_1,\theta_2,\omega))\right]$$

$$\text{s.t.}\quad\begin{cases} x_1(\theta_1,\theta_2,\omega)+x_2(\theta_1,\theta_2,\omega)=X(\omega) \qquad \forall\theta_1,\theta_2,\omega\\ \sum\limits_{\theta_2\in\Theta}\underline{\mu}\mu(\theta_2)\sum\limits_{\Omega}\pi(\underline{\theta},\theta_2,\omega)\left[u(x_1(\underline{\theta},\theta_2,\omega))-u(X_1(\omega))\right]\geq 0\\ \sum\limits_{\theta_1\in\Theta}\mu(\theta_1)\underline{\mu}\sum\limits_{\Omega}\pi(\theta_1,\underline{\theta},\omega)\left[u(x_2(\theta_1,\underline{\theta},\omega))-u(X_2(\omega))\right]\geq 0\\ \sum\limits_{\theta_2\in\Theta}\overline{\mu}\mu(\theta_2)\sum\limits_{\Omega}\pi(\overline{\theta},\theta_2,\omega)\left[u(x_1(\overline{\theta},\theta_2,\omega))-u(X_1(\omega))\right]\geq 0\\ \sum\limits_{\theta_1\in\Theta}\mu(\theta_1)\overline{\mu}\sum\limits_{\Omega}\pi(\theta_1,\overline{\theta},\omega)\left[u(x_2(\theta_1,\overline{\theta},\omega))-u(X_2(\omega))\right]\geq 0 \end{cases}$$

Letting the Lagrange multipliers be $\mu(\theta_1)\mu(\theta_2)\pi(\theta_1,\theta_2,\omega)\alpha\left(\theta_1,\theta_2,\omega\right), \underline{\gamma}, \underline{\gamma}, \overline{\gamma}$ and $\overline{\gamma}$ respectively [9], the fist order conditions are:

$$
\begin{aligned}
(1+\overline{\gamma})u'(x_1(\overline{\theta},\overline{\theta},\omega)) &= \alpha(\overline{\theta},\overline{\theta},\omega) && (2.10)\\
(1+\overline{\gamma})u'(x_2(\overline{\theta},\overline{\theta},\omega)) &= \alpha(\overline{\theta},\overline{\theta},\omega) && (2.11)\\
(1+\underline{\gamma})u'(x_1(\underline{\theta},\underline{\theta},\omega)) &= \alpha(\underline{\theta},\underline{\theta},\omega) && (2.12)\\
(1+\underline{\gamma})u'(x_2(\underline{\theta},\underline{\theta},\omega)) &= \alpha(\underline{\theta},\underline{\theta},\omega) && (2.13)\\
(1+\overline{\gamma})u'(x_1(\overline{\theta},\underline{\theta},\omega)) &= \alpha(\overline{\theta},\underline{\theta},\omega) && (2.14)\\
(1+\underline{\gamma})u'(x_2(\overline{\theta},\underline{\theta},\omega)) &= \alpha(\overline{\theta},\underline{\theta},\omega) && (2.15)\\
(1+\underline{\gamma})u'(x_1(\underline{\theta},\overline{\theta},\omega)) &= \alpha(\underline{\theta},\overline{\theta},\omega) && (2.16)\\
(1+\overline{\gamma})u'(x_2(\underline{\theta},\overline{\theta},\omega)) &= \alpha(\underline{\theta},\overline{\theta},\omega) && (2.17)
\end{aligned}
$$

Equations (2.10)–(2.13) together with the fist constraint gives $x_i(\theta,\theta,\omega) = X(\omega)/2$, $\theta = \overline{\theta}, \underline{\theta}$ and the IPC constraints become:

$$
\begin{aligned}
&\underline{\mu}\underline{\theta}(1-\underline{\theta})\left(2u(\widehat{x}) - u(\overline{x}) - u(\underline{x})\right) + \overline{\mu}\sum_{\Omega}\pi(\underline{\theta},\overline{\theta},\omega)\big(u(x_1(\underline{\theta},\overline{\theta},\omega)) - u(X_1(\omega))\big) \geq && (2.18)\\
&\overline{\mu}\overline{\theta}(1-\overline{\theta})\left(2u(\widehat{x}) - u(\overline{x}) - u(\underline{x})\right) + \underline{\mu}\sum_{\Omega}\pi(\underline{\theta},\overline{\theta},\omega)\big(u(x_2(\underline{\theta},\overline{\theta},\omega)) - u(X_2(\omega))\big) \geq && (2.19)
\end{aligned}
$$

Moreover by (2.16) and (2.17), $\dfrac{u'(x_1(\underline{\theta},\overline{\theta},\omega))}{u'(x_2(\underline{\theta},\overline{\theta},\omega))} = \dfrac{(1+\overline{\gamma})}{(1+\underline{\gamma})}$ which means together with $x_1(\underline{\theta},\overline{\theta},\omega) + x_2(\underline{\theta},\overline{\theta},\omega) = X(\omega)$ that $x_i(\underline{\theta},\overline{\theta},\omega)$ only depends on aggregate wealth and thus that $x_i(\underline{\theta},\overline{\theta},(a,b)) = x_i(\underline{\theta},\overline{\theta},(b,a)) \quad \forall i = 1,2;\ a,b \in \{0,1\}$.

[9] Because individuals are ex-ante identical, second and third, as well as forth and fifth constraints are the same

Now, assume $\underline{\gamma} < \overline{\gamma}$. It follows $x_1(\underline{\theta}, \overline{\theta}, \omega) < x_2(\underline{\theta}, \overline{\theta}, \omega)$ and thus: $x_2(\underline{\theta}, \overline{\theta}, \omega) > X(\omega)/2$ and $x_1(\underline{\theta}, \overline{\theta}, \omega) < X(\omega)/2$. By (2.18) and (2.19) this implies:

$$\begin{aligned} 0 &\leq \underline{\mu}\underline{\theta}(1-\underline{\theta})\left(2u(\widehat{x}) - u(\overline{x}) - u(\underline{x})\right) + \overline{\mu}\sum_{\Omega}\pi(\underline{\theta}, \overline{\theta}, \omega)\left(u(x_1(\underline{\theta}, \overline{\theta}, \omega)) - u(X_1(\omega)\right) \\ &< \underline{\theta}(1-\mathrm{E}(\theta))\left(u(\widehat{x}) - u(\underline{x})\right) + \mathrm{E}(\theta)\left(1-\underline{\theta}\right)\left(u(\widehat{x}) - u(\overline{x})\right) \end{aligned}$$

and

$$\begin{aligned} 0 &= \overline{\mu}\overline{\theta}(1-\overline{\theta})\left(2u(\widehat{x}) - u(\overline{x}) - u(\underline{x})\right) + \underline{\mu}\sum_{\Omega}\pi(\underline{\theta}, \overline{\theta}, \omega)\left(u(x_2(\underline{\theta}, \overline{\theta}, \omega)) - u(X_2(\omega)\right) \\ &> \overline{\theta}(1-\mathrm{E}(\theta))\left(u(\widehat{x}) - u(\underline{x})\right) + \mathrm{E}(\theta)\left(1-\overline{\theta}\right)\left(u(\widehat{x}) - u(\overline{x})\right) \end{aligned}$$

We thus end up with

$$\overline{\theta}(1-\mathrm{E}(\theta))\left(u(\widehat{x}) - u(\underline{x})\right) + \mathrm{E}(\theta)(1-\overline{\theta})\left(u(\widehat{x}) - u(\overline{x})\right) < 0 \leq \underline{\theta}(1-\mathrm{E}(\theta))\left(u(\widehat{x}) - u(\underline{x})\right) + \mathrm{E}(\theta)(1-\underline{\theta})\left(u(\widehat{x}) - u(\overline{x})\right)$$

which is in contradiction with the fact that $\dfrac{\overline{\theta}(1-\mathrm{E}(\theta))}{\mathrm{E}(\theta)(1-\overline{\theta})} \leq 1 \leq \dfrac{(1-\underline{\theta})\mathrm{E}(\theta)}{\underline{\theta}(1-\mathrm{E}(\theta))}$.

The optimum thus requires $\underline{\gamma} \geq \overline{\gamma}$, meaning that $x_1(\underline{\theta}, \overline{\theta}, \omega) \geq x_2(\underline{\theta}, \overline{\theta}, \omega) \quad \forall\omega$. This moreover implies that low risk IPC binds, that is:

$$\underline{\mu}\underline{\theta}(1-\underline{\theta})(2u(\widehat{x}) - u(\overline{x}) - u(\underline{x})) + \overline{\mu}\sum_{\Omega}\pi(\underline{\theta}, \overline{\theta}, \omega)\left[u(x_1(\underline{\theta}, \overline{\theta}, \omega)) - u(X_1(\omega))\right] = 0$$

2.6.2 Proof of Proposition 2.3

Under asymmetric information the program is:

$$\max_x \quad \sum_{\Theta^2} \mu(\theta_1)\mu(\theta_2) \sum_{\Omega} \pi(\theta_1, \theta_2, \omega) \left[u(x_1(\theta_1, \theta_2, \omega)) + u(x_2(\theta_1, \theta_2, \omega))\right] \tag{2.20}$$

$$\text{s.t.} \quad \begin{cases} x_1(\theta_1, \theta_2, \omega) + x_2(\theta_1, \theta_2, \omega) = X(\omega) \qquad \forall \theta_1, \theta_2, \omega \\ \sum_{\theta_2 \in \Theta} \mu(\theta_2) \sum_{\Omega} \pi(\overline{\theta}, \theta_2, \omega) \left[u(x_1(\overline{\theta}, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 \\ \sum_{\theta_2 \in \Theta} \mu(\theta_2) \sum_{\Omega} \pi(\overline{\theta}, \theta_2, \omega) \left(u(x_1(\overline{\theta}, \theta_2, \omega)) - u(x_1(\underline{\theta}, \theta_2, \omega))\right) \geq 0 \\ \sum_{\theta_2 \in \Theta} \mu(\theta_2) \sum_{\Omega} \pi(\underline{\theta}, \theta_2, \omega) \left[u(x_1(\underline{\theta}, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 \\ \sum_{\theta_2 \in \Theta} \mu(\theta_2) \sum_{\Omega} \pi(\underline{\theta}, \theta_2, \omega) \left(u(x_1(\underline{\theta}, \theta_2, \omega)) - u(x_1(\overline{\theta}, \theta_2, \omega))\right) \geq 0 \\ \sum_{\theta_1 \in \Theta} \mu(\theta_1) \sum_{\Omega} \pi(\theta_1, \overline{\theta}, \omega) \left[u(x_2(\theta_1, \overline{\theta}, \omega)) - u(X_2(\omega))\right] \geq 0 \\ \sum_{\theta_1 \in \Theta} \mu(\theta_1) \sum_{\Omega} \pi(\theta_1, \overline{\theta}, \omega) \left(u(x_2(\theta_1, \overline{\theta}, \omega)) - u(x_2(\theta_1, \underline{\theta}, \omega))\right) \geq 0 \\ \sum_{\theta_1 \in \Theta} \mu(\theta_1)\underline{\mu} \sum_{\Omega} \pi(\theta_1, \underline{\theta}, \omega) \left[u(x_2(\theta_1, \underline{\theta}, \omega)) - u(X_2(\omega))\right] \geq 0 \\ \sum_{\theta_1 \in \Theta} \mu(\theta_1)\underline{\mu} \sum_{\Omega} \pi(\theta_1, \underline{\theta}, \omega) \left(u(x_2(\theta_1, \underline{\theta}, \omega)) - u(x_2(\theta_1, \overline{\theta}, \omega))\right) \geq 0 \end{cases}$$

Let $\alpha(\theta_1, \theta_2, \omega)$, $\overline{\gamma}_1, \overline{\lambda}_1, \underline{\gamma}_1, \underline{\lambda}_1, \overline{\gamma}_2, \overline{\lambda}_2, \underline{\gamma}_2, \underline{\lambda}_2$ be the respective Lagrange multipliers.

Because individuals are assumed to be ex-ante identical, $x_1(\theta_1, \theta_2, (a, b)) = x_2(\theta_2, \theta_1, (b, a)) = X((b, a)) - x_1(\theta_2, \theta_1, (b, a)) \quad \forall a, b \in \{0, 1\}$ and thus $\alpha(\overline{\theta}, \underline{\theta}, (a, b)) = \alpha(\underline{\theta}, \overline{\theta}, (b, a))$, $\alpha(\overline{\theta}, \overline{\theta}, (1, 0)) = \alpha(\overline{\theta}, \overline{\theta}, (0, 1))$, $\alpha(\underline{\theta}, \underline{\theta}, (1, 0)) = \alpha(\underline{\theta}, \underline{\theta}, (0, 1))$, $\overline{\gamma}_1 = \overline{\gamma}_2 \equiv \overline{\gamma}$, $\overline{\lambda}_1 = \overline{\lambda}_2 \equiv \overline{\lambda}$, $\underline{\gamma}_1 = \underline{\gamma}_2 \equiv \underline{\gamma}$,$\underline{\lambda}_1 = \underline{\lambda}_2 \equiv \underline{\lambda}$

Lemma 2.1 *The optimum is unique*

Proof of Lemma 2.1

Letting $h_1(\theta_1, \theta_2, \omega) \equiv u(x_1(\theta_1, \theta_2, \omega)), h_2(\theta_1, \theta_2, \omega) \equiv u(x_2(\theta_1, \theta_2, \omega))$ and

$$h : \left| \begin{array}{c} \Theta^2 \times \Omega \to \mathbb{R}^2 \\ (\theta_1, \theta_2, \omega) \mapsto (h_1(\theta_1, \theta_2, \omega), h_2(\theta_1, \theta_2, \omega)) \end{array} \right.$$

the program becomes:

$$\max_h \quad \sum_{\Theta^2} \mu(\theta_1)\mu(\theta_2) \sum_{\Omega} \pi(\theta_1, \theta_2, \omega) \left[h_1(\theta_1, \theta_2, \omega)) + h_2(\theta_1, \theta_2, \omega))\right]$$

$$\text{s.t.} \quad \begin{cases} u^{-1}(h_1(\theta_1, \theta_2, \omega)) + u^{-1}(h_2(\theta_1, \theta_2, \omega)) = X(\omega) \qquad \forall \theta_1, \theta_2, \omega \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\overline{\theta}, \theta_2, \omega) \left[h_1(\overline{\theta}, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\overline{\theta}, \theta_2, \omega) \left(h_1(\overline{\theta}, \theta_2, \omega)) - h_1(\underline{\theta}, \theta_2, \omega))\right) \geq 0 \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\underline{\theta}, \theta_2, \omega) \left[h_1(\underline{\theta}, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 \\ \sum\limits_{\theta_2 \in \Theta} \mu(\theta_2) \sum\limits_{\Omega} \pi(\underline{\theta}, \theta_2, \omega) \left(h_1(\underline{\theta}, \theta_2, \omega)) - h_1(\overline{\theta}, \theta_2, \omega))\right) \geq 0 \\ \sum\limits_{\theta_1 \in \Theta} \mu(\theta_1) \sum\limits_{\Omega} \pi(\theta_1, \overline{\theta}, \omega) \left[h_2(\theta_1, \overline{\theta}, \omega)) - u(X_2(\omega))\right] \geq 0 \\ \sum\limits_{\theta_1 \in \Theta} \mu(\theta_1) \sum\limits_{\Omega} \pi(\theta_1, \overline{\theta}, \omega) \left(h_2(\theta_1, \overline{\theta}, \omega)) - h_2(\theta_1, \underline{\theta}, \omega))\right) \geq 0 \\ \sum\limits_{\theta_1 \in \Theta} \mu(\theta_1) \sum\limits_{\Omega} \pi(\theta_1, \underline{\theta}, \omega) \left[h_2(\theta_1, \underline{\theta}, \omega)) - u(X_2(\omega))\right] \geq 0 \\ \sum\limits_{\theta_1 \in \Theta} \mu(\theta_1) \sum\limits_{\Omega} \pi(\theta_1, \underline{\theta}, \omega) \left(h_2(\theta_1, \underline{\theta}, \omega)) - h_2(\theta_1, \overline{\theta}, \omega))\right) \geq 0 \end{cases}$$

In $h_1(.)$ and $h_2(.)$ we then have one strictly convex equality constraints and multiple linear inequality constraint. This defines a strictly convex constraint set. Since the gradient of the linear objective is not equal to the gradient of any linear constraint, the optimum must be unique.

Lemma 2.2 *The mutuality principle is not sustainable and the optimal sharing rule implies exchange in some states where initial wealth are identical. Therefore, autarky is never optimal.*

Proof of Lemma 2.2

The first order conditions of (2.20) can then be written as:

$$\begin{cases} \left[1+\overline{\gamma}+\overline{\lambda}-\frac{\underline{\mu}\pi(\underline{\theta},\theta_2,\omega)}{\overline{\mu}\pi(\overline{\theta},\theta_2,\omega)}\underline{\lambda}\right] u'(x_1(\overline{\theta},\theta_2,\omega)) = \alpha(\overline{\theta},\theta_2,\omega) \quad \forall\theta_2,\omega \\ \left[1+\overline{\gamma}+\overline{\lambda}-\frac{\underline{\mu}\pi(\theta_1,\underline{\theta},\omega)}{\overline{\mu}\pi(\theta_1,\overline{\theta},\omega)}\underline{\lambda}\right] u'(x_2(\theta_1,\overline{\theta},\omega)) = \alpha(\theta_1,\overline{\theta},\omega) \quad \forall\theta_1,\omega \\ \left[1+\underline{\gamma}+\underline{\lambda}-\frac{\overline{\mu}\pi(\overline{\theta},\theta_2,\omega)}{\underline{\mu}\pi(\underline{\theta},\theta_2,\omega)}\overline{\lambda}\right] u'(x_1(\underline{\theta},\theta_2,\omega)) = \alpha(\underline{\theta},\theta_2,\omega) \quad \forall\theta_2,\omega \\ \left[1+\underline{\gamma}+\underline{\lambda}-\frac{\overline{\mu}\pi(\theta_1,\underline{\theta},\omega)}{\underline{\mu}\pi(\theta_1,\underline{\theta},\omega)}\overline{\lambda}\right] u'(x_2(\theta_1,\underline{\theta},\omega)) = \alpha(\theta_1,\underline{\theta},\omega) \quad \forall\theta_1,\omega \end{cases}$$

First of all, when both individual announce the same type and have the same initial wealth, ex-ante anonymity implies : $x_1(\theta,\theta,(0,0)) = x_2(\theta,\theta,(0,0)) = \overline{x}$ and $x_1(\theta,\theta,(1,1)) = x_2(\theta,\theta,(1,1)) = \underline{x}$, $\forall\theta \in \underline{\theta},\overline{\theta}$ (note that this is fortunately confirmed by the first order conditions).

Now, when agents announce the same risk but have different initial wealth, the first order conditions lead to:

$$\begin{aligned} \frac{u'(x_2(\overline{\theta},\overline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\overline{\theta},(1,0)))} &= \frac{\left(1+\overline{\gamma}+\overline{\lambda}-\frac{\underline{\theta}}{\overline{\theta}}\underline{\mu}\underline{\lambda}\right)}{\left(1+\overline{\gamma}+\overline{\lambda}-\frac{(1-\underline{\theta})}{(1-\overline{\theta})}\underline{\mu}\underline{\lambda}\right)} \equiv \frac{A}{B} \\ \frac{u'(x_2(\underline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\underline{\theta},\underline{\theta},(1,0)))} &= \frac{\left(1+\underline{\gamma}+\underline{\lambda}-\frac{\overline{\theta}}{\underline{\theta}}\overline{\mu}\overline{\lambda}\right)}{\left(1+\underline{\gamma}+\underline{\lambda}-\frac{(1-\overline{\theta})}{(1-\underline{\theta})}\overline{\mu}\overline{\lambda}\right)} \equiv \frac{C}{D} \end{aligned}$$

As $\underline{\theta} < \overline{\theta}$, we have $A \geq B$ and $D \geq C$. The optimal sharing rule has thus to satisfy:

$$x_1(\overline{\theta},\overline{\theta},(1,0)) \geq x_2(\overline{\theta},\overline{\theta},(1,0)) \quad (\underline{\lambda}) \tag{2.21}$$

$$x_2(\underline{\theta},\underline{\theta},(1,0)) \geq x_1(\underline{\theta},\underline{\theta},(1,0)) \quad (\overline{\lambda}) \tag{2.22}$$

The Lagrange multiplier in bracket is the one that have to be null for the corresponding equation to be satisfied with equality.

The mutuality principle would imply in this setting that $x_1(\theta_1, \theta_2, (0,1)) = x_1(\theta_1, \theta_2, (1,0))$ and notably that:

- for $\theta_1 = \theta_2 = \overline{\theta}$, $x_1(\overline{\theta}, \overline{\theta}, (1,0)) = x_1(\overline{\theta}, \overline{\theta}, (0,1)) = x_2(\overline{\theta}, \overline{\theta}, (0,1))$ which would lead to $\underline{\lambda} = 0$ by (2.21)
- for $\theta_1 = \theta_2 = \underline{\theta}$, $x_1(\underline{\theta}, \underline{\theta}, (1,0)) = x_1(\underline{\theta}, \underline{\theta}, (0,1)) = x_2(\underline{\theta}, \underline{\theta}, (0,1))$ which would lead to $\overline{\lambda} = 0$ by (2.22)

The mutuality principle would then be sustainable only if the complete information allocation were Bayesian incentive compatible for both type of individuals $\big(\underline{\lambda} = \overline{\lambda} = 0\big)$. Thus, the mutuality principle is not sustainable when $\dfrac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} \geq \dfrac{(1-\underline{\theta})\mathrm{E}(\theta)}{\underline{\theta}(1-\mathrm{E}(\theta))}$.

Finally when agents announce different types, the solution may be written as:

$$\begin{aligned}
\frac{u'(x_2(\overline{\theta}, \underline{\theta}, (0,0))}{u'(x_1(\overline{\theta}, \underline{\theta}, (0,0)))} &= \frac{\left(1 + \overline{\gamma} + \overline{\lambda} - \frac{(1-\underline{\theta})}{(1-\overline{\theta})}\underline{\mu}\underline{\lambda}\right)}{\left(1 + \underline{\gamma} + \underline{\lambda} - \frac{(1-\overline{\theta})}{(1-\underline{\theta})}\overline{\mu}\overline{\lambda}\right)} = \frac{B}{D} \\
\frac{u'(x_2(\overline{\theta}, \underline{\theta}, (1,0)))}{u'(x_1(\overline{\theta}, \underline{\theta}, (1,0)))} &= \frac{\left(1 + \overline{\gamma} + \overline{\lambda} - \frac{\underline{\theta}}{\overline{\theta}}\underline{\mu}\underline{\lambda}\right)}{\left(1 + \underline{\gamma} + \underline{\lambda} - \frac{(1-\overline{\theta})}{(1-\underline{\theta})}\overline{\mu}\overline{\lambda}\right)} = \frac{A}{D} \\
\frac{u'(x_2(\overline{\theta}, \underline{\theta}, (0,1)))}{u'(x_1(\overline{\theta}, \underline{\theta}, (0,1)))} &= \frac{\left(1 + \overline{\gamma} + \overline{\lambda} - \frac{(1-\underline{\theta})}{(1-\overline{\theta})}\underline{\mu}\underline{\lambda}\right)}{\left(1 + \underline{\gamma} + \underline{\lambda} - \frac{\overline{\theta}}{\underline{\theta}}\overline{\mu}\overline{\lambda}\right)} = \frac{B}{C} \\
\frac{u'(x_2(\overline{\theta}, \underline{\theta}, (1,1)))}{u'(x_1(\overline{\theta}, \underline{\theta}, (1,1))} &= \frac{\left(1 + \overline{\gamma} + \overline{\lambda} - \frac{\underline{\theta}}{\overline{\theta}}\underline{\mu}\underline{\lambda}\right)}{\left(1 + \underline{\gamma} + \underline{\lambda} - \frac{\overline{\theta}}{\underline{\theta}}\overline{\mu}\overline{\lambda}\right)} = \frac{A}{C}
\end{aligned}$$

As $B \leq A, D \leq C$ the following inequalities hold:

$$\frac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} = \underbrace{\frac{B}{D} \leq \frac{A}{D}}_{(\underline{\lambda})} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))} = \underbrace{\frac{A}{D} \leq \frac{A}{C}}_{(\overline{\lambda})} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} \quad (2.23)$$

$$\frac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} = \underbrace{\frac{B}{D} \leq \frac{B}{C}}_{(\overline{\lambda})} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))} = \underbrace{\frac{B}{C} \leq \frac{A}{C}}_{(\underline{\lambda})} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} \quad (2.24)$$

If there were no exchange when initial wealths are identical, that is if $x_1(\overline{\theta},\underline{\theta},(0,0)) = x_2(\overline{\theta},\underline{\theta},(0,0))) = \underline{x}$ and $x_1(\overline{\theta},\underline{\theta},(1,1)) = x_2(\overline{\theta},\underline{\theta},(1,1))) = \overline{x}$, the six previous ratios would be equal to one. As it implies $\underline{\lambda} = \overline{\lambda} = 0$, what has been shown to be impossible when heterogeneity is high. This imply in particular that autarky is not optimal and the optimal sharing rule calls for exchange in some states where initial wealth are identical.

2.6.3 Proof of Proposition 2.4

Lemma 2.3 *The participation constraint for low risk individual necessarily binds whereas the one of high risk is always strictly satisfied at the optimum*

Proof of Lemma 2.3

- If both participation constraints were binding, that is if $\overline{\gamma}$ and $\underline{\gamma}$ were both positive, by construction, the utilitarian expected utility achieved by autarky would be optimal. This has been shown to be impossible, by unicity of the optimum.Thus one participation constraint necessarily does not bind.
- The first best allocation, that has been proven not to be optimal when $\dfrac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} \geq \dfrac{(1-\underline{\theta})\mathrm{E}(\theta)}{\underline{\theta}(1-\mathrm{E}(\theta))}$, satisfies the low risk individual constraint but not the one of high risk individual.

Therefore $\overline{\gamma} = 0$ and $\underline{\gamma} > 0$

Then,

$$
\begin{aligned}
B &= \left(1+\overline{\lambda}-\frac{(1-\underline{\theta})}{(1-\overline{\theta})}\underline{\mu}\underline{\lambda}\right)\\
D &= \left(1+\underline{\gamma}+\underline{\lambda}-\frac{(1-\overline{\theta})}{(1-\underline{\theta})}\overline{\mu}\overline{\lambda}\right)\\
A &= \left(1+\overline{\lambda}-\frac{\underline{\theta}}{\overline{\theta}}\underline{\mu}\underline{\lambda}\right)\\
C &= \left(1+\underline{\gamma}+\underline{\lambda}-\frac{\overline{\theta}}{\underline{\theta}}\overline{\mu}\overline{\lambda}\right)
\end{aligned}
$$

Lemma 2.4 *The Bayesian Incentive constraint for high risk individuals necessarily binds.*

Proof of Lemma 2.4

Let set :

$$
\underline{\pi} \equiv \begin{pmatrix}
\underline{\mu}(1-\underline{\theta})(1-\underline{\theta})\\
\underline{\mu}\underline{\theta}(1-\underline{\theta})\\
\underline{\mu}(1-\underline{\theta})\underline{\theta}\\
\underline{\mu}\underline{\theta}\underline{\theta}\\
\overline{\mu}(1-\underline{\theta})(1-\overline{\theta})\\
\overline{\mu}\underline{\theta}(1-\overline{\theta})\\
\overline{\mu}(1-\underline{\theta})\overline{\theta}\\
\overline{\mu}\underline{\theta}\overline{\theta}
\end{pmatrix}, \overline{\pi} \equiv \begin{pmatrix}
\underline{\mu}(1-\overline{\theta})(1-\underline{\theta})\\
\underline{\mu}\overline{\theta}(1-\underline{\theta})\\
\underline{\mu}(1-\overline{\theta})\underline{\theta}\\
\underline{\mu}\overline{\theta}\underline{\theta}\\
\overline{\mu}(1-\overline{\theta})(1-\overline{\theta})\\
\overline{\mu}\overline{\theta}(1-\overline{\theta})\\
\overline{\mu}(1-\overline{\theta})\overline{\theta}\\
\overline{\mu}\overline{\theta}\overline{\theta}
\end{pmatrix}, \delta \equiv \begin{pmatrix}
u(x_1(\underline{\theta},\underline{\theta},(0,0)))-u(x_1(\overline{\theta},\underline{\theta},(0,0)))\\
u(x_1(\underline{\theta},\underline{\theta},(1,0)))-u(x_1(\overline{\theta},\underline{\theta},(1,0)))\\
u(x_1(\underline{\theta},\underline{\theta},(0,1)))-u(x_1(\overline{\theta},\underline{\theta},(0,1)))\\
u(x_1(\underline{\theta},\underline{\theta},(1,1)))-u(x_1(\overline{\theta},\underline{\theta},(1,1)))\\
u(x_1(\underline{\theta},\overline{\theta},(0,0)))-u(x_1(\overline{\theta},\overline{\theta},(0,0)))\\
u(x_1(\underline{\theta},\overline{\theta},(1,0)))-u(x_1(\overline{\theta},\overline{\theta},(1,0)))\\
u(x_1(\underline{\theta},\overline{\theta},(0,1)))-u(x_1(\overline{\theta},\overline{\theta},(0,1)))\\
u(x_1(\underline{\theta},\overline{\theta},(1,1)))-u(x_1(\overline{\theta},\overline{\theta},(1,1)))
\end{pmatrix}
$$

$$\underline{v} \equiv \begin{pmatrix} u(x_1(\underline{\theta},\underline{\theta},(0,0))) - u(\overline{x}) \\ u(x_1(\underline{\theta},\underline{\theta},(1,0))) - u(\underline{x}) \\ u(x_1(\underline{\theta},\underline{\theta},(0,1))) - u(\overline{x}) \\ u(x_1(\underline{\theta},\underline{\theta},(1,1))) - u(\underline{x}) \\ u(x_1(\underline{\theta},\overline{\theta},(0,0))) - u(\overline{x}) \\ u(x_1(\underline{\theta},\overline{\theta},(1,0))) - u(\underline{x}) \\ u(x_1(\underline{\theta},\overline{\theta},(0,1))) - u(\overline{x}) \\ u(x_1(\underline{\theta},\overline{\theta},(1,1))) - u(\underline{x}) \end{pmatrix}, \overline{v} \equiv \begin{pmatrix} u(\overline{x}) - u(x_1(\overline{\theta},\underline{\theta},(0,0))) \\ u(\underline{x}) - u(x_1(\overline{\theta},\underline{\theta},(1,0))) \\ u(\overline{x}) - u(x_1(\overline{\theta},\underline{\theta},(0,1))) \\ u(\underline{x}) - u(x_1(\overline{\theta},\underline{\theta},(1,1))) \\ u(\overline{x}) - u(x_1(\overline{\theta},\overline{\theta},(0,0))) \\ u(\underline{x}) - u(x_1(\overline{\theta},\overline{\theta},(1,0))) \\ u(\overline{x}) - u(x_1(\overline{\theta},\overline{\theta},(0,1))) \\ u(\overline{x}) - u(x_1(\overline{\theta},\overline{\theta},(1,1))) \end{pmatrix}$$

the constraints become respectively:

- Bayesian Incentive constraint for low risk: $\underline{\pi}.\delta \geq 0$
- Bayesian Incentive constraint for high risk: $\overline{\pi}.\delta \leq 0$
- Participation constraint for low risk: $\underline{\pi}.\underline{v} = 0$
- Participation constraint for high risk: $\overline{\pi}.\overline{v} < 0$

Moreover one has $\underline{v} + \overline{v} = \delta$.

Supposing $\overline{\lambda} = 0$ then would lead by (2.22) to $x_1(\underline{\theta},\underline{\theta},(1,0)) = x_1(\underline{\theta},\underline{\theta},(0,1))$.

As $\overline{\lambda} = 0$ and $\underline{\lambda} = 0$ can not be simultaneously null, it follows $\overline{\lambda} \neq 0$, whicht implies:

* $\underline{\pi}.\delta = 0$ and thus $\underline{\pi}.\underline{v} + \underline{\pi}.\overline{v} = 0$. And hence, since $\underline{\pi}.\underline{v} = 0$, $\underline{\pi}.\overline{v} = 0$.

* by (2.23) and (2.24) it follows:

$$\frac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))} = \frac{B}{C} < \frac{A}{C} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))}$$

With $B = \left(1 - \frac{\underline{\mu}(1-\underline{\theta})}{\mu(1-\overline{\theta})}\underline{\lambda}\right)$, $A = \left(1 - \frac{\underline{\mu}\underline{\theta}}{\mu\overline{\theta}}\underline{\lambda}\right)$, $C = 1 + \underline{\gamma} + \underline{\lambda}$

We necessarily would have:

$$\frac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x1(\overline{\theta},\underline{\theta},(0,0)))}=\frac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))}<\frac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))}=\frac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))}<1$$

That implies :

$$\begin{cases} x_1(\overline{\theta},\underline{\theta},\omega)<\frac{X(\omega)}{2}<x_2(\overline{\theta},\underline{\theta},\omega) \\ x_1(\underline{\theta},\overline{\theta},\omega)>\frac{X(\omega)}{2}>x_2(\underline{\theta},\overline{\theta},\omega) \end{cases}$$

Recalling $x_1(\theta,\theta,\omega)\leq\frac{X(\omega)}{2}\leq x_2(\theta,\theta,\omega)$ we have :

$$\underline{\pi}.\overline{v}=\underline{\pi}\bullet\begin{pmatrix} u(\overline{x})-u(x_1(\overline{\theta},\underline{\theta},(0,0))) \\ u(\underline{x})-u(x_1(\overline{\theta},\underline{\theta},(1,0))) \\ u(\overline{x})-u(x_1(\overline{\theta},\underline{\theta},(0,1))) \\ u(\underline{x})-u(x_1(\overline{\theta},\underline{\theta},(1,1))) \\ u(\overline{x})-u(x_1(\overline{\theta},\overline{\theta},(0,0))) \\ u(\underline{x})-u(x_1(\overline{\theta},\overline{\theta},(1,0))) \\ u(\overline{x})-u(x_1(\overline{\theta},\overline{\theta},(0,1))) \\ u(\overline{x})-u(x_1(\overline{\theta},\overline{\theta},(1,1))) \end{pmatrix}=\underline{\pi}\bullet\begin{pmatrix} u(\overline{x})-u(x_1(\overline{\theta},\underline{\theta},(0,0))) \\ u(\underline{x})-u(\widehat{x})+u(\widehat{x})-(x_1(\overline{\theta},\underline{\theta},(1,0))) \\ u(\overline{x})-u(\widehat{x})+u(\widehat{x})-u(x_1(\overline{\theta},\underline{\theta},(0,1))) \\ u(\underline{x})-u(x_1(\overline{\theta},\underline{\theta},(1,1))) \\ u(\overline{x})-u(x_1(\overline{\theta},\overline{\theta},(0,0))) \\ u(\underline{x})-u(\widehat{x})+u(\widehat{x})-u(x_1(\overline{\theta},\overline{\theta},(1,0))) \\ u(\overline{x})-u(\widehat{x})+u(\widehat{x})-u(x_1(\overline{\theta},\overline{\theta},(0,1))) \\ u(\overline{x})-u(x_1(\overline{\theta},\overline{\theta},(1,1))) \end{pmatrix}$$

We hence would have :

$$\underline{\pi}.\overline{v}>\begin{pmatrix} \underline{\mu}(1-\underline{\theta})(1-\underline{\theta}) \\ \underline{\mu}\underline{\theta}(1-\underline{\theta}) \\ \underline{\mu}(1-\underline{\theta})\underline{\theta} \\ \underline{\mu\theta\theta} \\ \overline{\mu}(1-\underline{\theta})(1-\overline{\theta}) \\ \overline{\mu}\underline{\theta}(1-\overline{\theta}) \\ \overline{\mu}(1-\underline{\theta})\overline{\theta} \\ \overline{\mu}\underline{\theta}\overline{\theta} \end{pmatrix}\bullet\begin{pmatrix} 0 \\ u(\underline{x})-u(\widehat{x}) \\ u(\overline{x})-u(\widehat{x}) \\ 0 \\ 0 \\ u(\underline{x})-u(\widehat{x}) \\ u(\overline{x})-u(\widehat{x}) \\ 0 \end{pmatrix}$$

The right hand side of the previous inequality is equal to $[\underline{\theta}(1-\mathrm{E}(\theta))\left(u(\underline{x})-u(\widehat{x})\right)+(1-\underline{\theta})\mathrm{E}\left(\theta\right)\left(u(\overline{x})-\right.$ and is therefore positive when $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\leq\dfrac{(1-\underline{\theta})\mathrm{E}(\theta)}{\underline{\theta}(1-\mathrm{E}(\theta))}$.

We thus end up with a contradiction meaning that we necessarily have $\overline{\lambda}>0$. Therefore, the Bayesian incentive constraint for high risk individuals binds at the optimum.

Lemma 2.5 *If preferences are HARA with $\gamma\geq\frac{1}{2}$ then, when $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\leq\dfrac{(1-\underline{\theta})E(\theta)}{\underline{\theta}(1-E(\theta))}$:*

$$\begin{cases}
x_i(\overline{\theta},\overline{\theta},\omega) & = & \frac{X(\omega)}{2} \\
x_1(\underline{\theta},\underline{\theta},(0,1)) & > & \widehat{x} \\
\nu_1\equiv\dfrac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} & = & \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))}<1 \\
\nu_2\equiv\dfrac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} & = & \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))}>1
\end{cases}$$

Proof of Lemma 2.5

From lemma 2.4 we know that $\overline{\lambda}>0$. We are going to show that there exists a solution of the optimization program with $\underline{\lambda}=0$. As we know that the solution is unique, this will give the result.

We set hence $\overline{\lambda}>0$, $\underline{\lambda}=0$, $\overline{\gamma}=0$ and $\underline{\gamma}>0$, which give :

$$\begin{aligned}
A &= B=\left(1+\overline{\lambda}\right) \\
D &= \left(1+\underline{\gamma}-\frac{(1-\overline{\theta})}{(1-\underline{\theta})}\overline{\mu}\overline{\lambda}\right) \\
C &= \left(1+\underline{\gamma}-\frac{\overline{\theta}}{\underline{\theta}}\overline{\mu}\overline{\lambda}\right)
\end{aligned}$$

By setting $\nu_1 \equiv \frac{A}{D}$ and $\nu_2 \equiv \frac{A}{C}$ the first order conditions give :

$$\left\{ \begin{array}{rcl} \dfrac{u'(x_2(\overline{\theta},\overline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\overline{\theta},(1,0)))} & = & 1 \\ \dfrac{u'(x_2(\underline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\underline{\theta},\underline{\theta},(1,0)))} & = & \frac{\nu_1}{\nu_2} < 1 \\ \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} & = & \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))} = \nu_1 < \nu_2 = \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))} = \dfrac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} \end{array} \right.$$

Let us first define the function $\varphi(\cdot,\cdot)$ as: $\dfrac{u'(2X-\varphi(X,\nu))}{u'(\varphi(X,\nu))} = \nu$.

It is easy to show that $\varphi(X,\nu)$ is a strictly increasing function with X and ν, with $\varphi(X,1) = X$ and $2X - \varphi(X,\nu) = \varphi\left(X, \frac{1}{\nu}\right)$.

Moreover simple calculations give (we note φ_ν and φ_X the partial derivatives of φ and $A(Y)$ the index of absolute aversion of u at Y):

$$\begin{array}{rcl} \nu\varphi_\nu(X,\nu) & = & \dfrac{1}{A(\varphi\left(X,\frac{1}{\nu}\right)) + A\left(\varphi(X,\nu)\right)} \\ \varphi_X(X,\nu) & = & \dfrac{2A(\varphi\left(X,\frac{1}{\nu}\right))}{A(\varphi\left(X,\frac{1}{\nu}\right)) + A(\varphi(X,\nu))} \end{array}$$

For HARA functions φ can be put on the following form :

$$\varphi(X,\nu) = \frac{2X\nu^{1/\gamma} - (1-\nu^{1/\gamma})\gamma\eta}{1+\nu^{1/\gamma}}$$

and:

$$u\left(\varphi(X,\nu)\right) = u(X)\left(\frac{2\nu^{1/\gamma}}{\left(1+\nu^{1/\gamma}\right)}\right)^{1-\gamma}$$

Using this function we have:

$$
\begin{aligned}
x_1(\overline{\theta}, \underline{\theta}, (0,0)) &= \varphi(\overline{x}, \nu_1) \\
x_1(\overline{\theta}, \underline{\theta}, (1,0)) &= \varphi(\widehat{x}, \nu_1) \\
x_1(\overline{\theta}, \underline{\theta}, (0,1)) &= \varphi(\widehat{x}, \nu_2) \\
x_1(\overline{\theta}, \underline{\theta}, (1,1)) &= \varphi(\underline{x}, \nu_2) \\
x_1(\underline{\theta}, \underline{\theta}, (1,0)) &= \varphi\left(\widehat{x}, \frac{\nu_1}{\nu_2}\right) \\
x_1(\underline{\theta}, \underline{\theta}, (a,a)) &= \frac{X(\omega)}{2} \\
x_1(\overline{\theta}, \overline{\theta}, \omega) &= \frac{X(\omega)}{2}
\end{aligned}
$$

Then,

$$
\underline{v} \equiv \begin{pmatrix}
0 \\
u\left(\varphi\left(\widehat{x}, \frac{\nu_1}{\nu_2}\right)\right) - u(\underline{x}) \\
u\left(2\widehat{x} - \varphi\left(\widehat{x}, \frac{\nu_1}{\nu_2}\right)\right) - u(\overline{x}) \\
0 \\
u\left(2\overline{x} - \varphi(\overline{x}, \nu_1)\right) - u(\overline{x}) \\
u\left(2\widehat{x} - \varphi(\widehat{x}, \nu_2)\right) - u(\underline{x}) \\
u\left(2\widehat{x} - \varphi(\widehat{x}, \nu_1)\right) - u(\overline{x}) \\
u\left(2\underline{x} - \varphi(\underline{x}, \nu_2)\right) - u(\underline{x})
\end{pmatrix}, \overline{v} \equiv \begin{pmatrix}
u(\overline{x}) - u\left(\varphi(\overline{x}, \nu_1)\right) \\
u(\underline{x}) - u\left(\varphi(\widehat{x}, \nu_1)\right) \\
u(\overline{x}) - u\left(\varphi(\widehat{x}, \nu_2)\right) \\
u(\underline{x}) - u\left(\varphi(\underline{x}, \nu_2)\right) \\
0 \\
u(\underline{x}) - u(\widehat{x}) \\
u(\overline{x}) - u(\widehat{x}) \\
0
\end{pmatrix}
$$

and:

$$\delta \equiv \begin{pmatrix} u(\overline{x}) - u\left(\varphi(\overline{x}, \nu_1)\right) \\ u\left(\varphi\left(\widehat{x}, \frac{\nu_1}{\nu_2}\right)\right) - u\left(\varphi(\widehat{x}, \nu_1)\right) \\ u\left(\varphi\left(\widehat{x}, \frac{\nu_2}{\nu_1}\right)\right) - u\left(\varphi(\widehat{x}, \nu_2)\right) \\ u(\underline{x}) - u\left(\varphi(\underline{x}, \nu_2)\right) \\ u\left(2\overline{x} - \varphi(\overline{x}, \nu_1)\right) - u(\overline{x}) \\ u\left(2\widehat{x} - \varphi(\widehat{x}, \nu_2)\right) - u(\widehat{x}) \\ u\left(2\widehat{x} - \varphi(\widehat{x}, \nu_1)\right) - u(\widehat{x}) \\ u\left(2\underline{x} - \varphi(\underline{x}, \nu_2)\right) - u(\underline{x}) \end{pmatrix}$$

Setting

$$\pi(\theta) \equiv \begin{pmatrix} (1-\theta)\underline{\mu}(1-\underline{\theta}) \\ \theta\underline{\mu}(1-\underline{\theta}) \\ (1-\theta)\underline{\mu}\underline{\theta} \\ \theta\underline{\mu}\underline{\theta} \\ (1-\theta)\overline{\mu}(1-\overline{\theta}) \\ \theta\overline{\mu}(1-\overline{\theta}) \\ (1-\theta)\overline{\mu}\overline{\theta} \\ \theta\overline{\mu}\overline{\theta} \end{pmatrix}$$

we can write $\pi(\theta) \bullet \delta$ as:

$$\pi(\theta) \bullet \delta = (1-\theta)\left[\underline{\mu}P + \overline{\mu}Q\right] + \theta\left[\underline{\mu}R + \overline{\mu}S\right]$$

with

$$\begin{aligned}
P &= (1-\underline{\theta})\left(u(\overline{x}) - u\left(\varphi(\overline{x},\nu_1)\right)\right) + \underline{\theta}\left(u\left(\varphi\left(\widehat{x},\frac{\nu_2}{\nu_1}\right)\right) - u\left(\varphi(\widehat{x},\nu_2)\right)\right)\\
Q &= (1-\overline{\theta})\left(u\left(2\overline{x} - \varphi(\overline{x},\nu_1)\right) - u(\overline{x})\right) + \overline{\theta}\left(u\left(2\widehat{x} - \varphi(\widehat{x},\nu_1)\right) - u(\widehat{x})\right)\\
R &= (1-\underline{\theta})\left(u\left(\varphi\left(\widehat{x},\frac{\nu_1}{\nu_2}\right)\right) - u\left(\varphi(\widehat{x},\nu_1)\right)\right) + \underline{\theta}\left(u(\underline{x}) - u\left(\varphi(\underline{x},\nu_2)\right)\right)\\
S &= (1-\overline{\theta})\left(u\left(2\widehat{x} - \varphi(\widehat{x},\nu_2)\right) - u(\widehat{x})\right) + \overline{\theta}\left(u\left(2\underline{x} - \varphi(\underline{x},\nu_2)\right) - u(\underline{x})\right)
\end{aligned}$$

$\pi(\theta)\bullet\delta = 0$ then represents the equation of the BIC(θ) constraint in the plane (ν_1,ν_2).

In the following, we prove that BIC($\underline{\theta}$) and BIC($\overline{\theta}$) only cross once (anti-clockwise) in this plane (at the point $\nu_1=\nu_2=1$).

Noting $F_i = \frac{\partial F}{\partial \nu_i}$ and $F_X = \frac{\partial F}{\partial X}$,

$$\frac{\partial\left(\pi(\theta)\bullet\delta\right)}{\partial\nu_i} = (1-\theta)\left[\underline{\mu}P_i + \overline{\mu}Q_i\right] + \theta\left[\underline{\mu}R_i + \overline{\mu}S_i\right]$$

Therefore

$$\Omega = \frac{\frac{\partial(\pi(\theta)\bullet\delta)}{\partial\nu_1}}{\frac{\partial(\pi(\theta)\bullet\delta)}{\partial\nu_2}} = \frac{(1-\theta)\left[\underline{\mu}P_1 + \overline{\mu}Q_1\right] + \theta\left[\underline{\mu}R_1\right]}{(1-\theta)\left[\underline{\mu}P_2\right] + \theta\left[\underline{\mu}R_2 + \overline{\mu}S_2\right]}$$

.

The sign of $\frac{\partial\Omega}{\partial\theta}$ is then the sign of:

$$\left[\underline{\mu}P_2\right]\left[\underline{\mu}R_1\right] - \left[\underline{\mu}P_1 + \overline{\mu}Q_1\right]\left[\underline{\mu}R_2 + \overline{\mu}S_2\right]$$

that is the sign of:

$$\underline{\mu}^2\left[R_1P_2 - P_1R_2\right] - \underline{\mu}\overline{\mu}\left[P_1S_2 + Q_1R_2\right] - \overline{\mu}^2 Q_1S_2$$

As $P_1 \leq 0$, $Q_1 \leq 0$, $S_2 \leq 0$, $R_2 \leq 0$, the sign is negative whatever $\underline{\mu}$ if and only if $R_1P_2 \leq P_1R_2$

Letting

$$\begin{aligned} P &= (1-\underline{\theta})\alpha(\overline{x},\nu_1) + \underline{\theta}\beta(\widehat{x},\nu_1,\nu_2) \\ R &= (1-\underline{\theta})\gamma(\widehat{x},\nu_1,\nu_2) + \underline{\theta}\delta(\underline{x},\nu_2) \end{aligned}$$

we obtain:

$$\begin{aligned} R_1P_2 - P_1R_2 &= (1-\underline{\theta})\underline{\theta}\gamma_1\beta_2 - ((1-\underline{\theta})\alpha_1 + \underline{\theta}\beta_1)\left((1-\underline{\theta})\gamma_2 + \underline{\theta}\delta_2\right) \\ &= (1-\underline{\theta})\underline{\theta}\left(\gamma_1\beta_2 - \gamma_2\beta_1 - \alpha_1\delta_2\right) - (1-\underline{\theta})^2\alpha_1\gamma_2 - \underline{\theta}^2\beta_1\delta_2 \end{aligned}$$

with $\alpha_1 \leq 0$, $\beta_1 \leq 0$ $\gamma_2 \leq 0$ $\delta_2 \leq 0$

Therefore $R_1P_2 - P_1R_2$ is negative whatever $\underline{\theta}$ if and only if $\gamma_1\beta_2 - \gamma_2\beta_1 - \alpha_1\delta_2 \leq 0$.

After some tedious computation this amounts to:

$$\begin{aligned} -\left[u\left(\varphi\left(\widehat{x},\frac{\nu_1}{\nu_2}\right)\right)\right]_1 \quad \left[u\left(\varphi(\widehat{x},\nu_2)\right)\right]_2 - \left[u\left(\varphi(\widehat{x},\nu_1)\right)\right]_1 \left[u\left(\varphi\left(\widehat{x},\tfrac{\nu_2}{\nu_1}\right)\right)\right]_2 \\ +\left[u\left(\varphi(\widehat{x},\nu_1)\right)\right]_1 \quad \left[u\left(\varphi(\widehat{x},\nu_2)\right)\right]_2 - \left[u\left(\varphi(\overline{x},\nu_1)\right)\right]_1 \left[u\left(\varphi(\underline{x},\nu_2)\right)\right]_2 \quad \leq 0 \end{aligned}$$

For which a sufficient condition is :

$$\frac{\left[u\left(\varphi\left(\widehat{x},\frac{\nu_1}{\nu_2}\right)\right)\right]_1}{\left[u\left(\varphi(\widehat{x},\nu_1)\right)\right]_1} + \frac{\left[u\left(\varphi\left(\widehat{x},\frac{\nu_2}{\nu_1}\right)\right)\right]_2}{\left[u\left(\varphi(\widehat{x},\nu_2)\right)\right]_2} \geq 1$$

That is :

$$\frac{1}{\nu_2}\frac{u'\left(\varphi\left(\widehat{x},\frac{\nu_1}{\nu_2}\right)\right)\varphi_t\left(\widehat{x},\frac{\nu_1}{\nu_2}\right)}{u'\left(\varphi\left(\widehat{x},\nu_1\right)\right)\varphi_t\left(\widehat{x},\nu_1\right)} + \frac{1}{\nu_1}\frac{u'\left(\varphi\left(\widehat{x},\frac{\nu_2}{\nu_1}\right)\right)\varphi_\nu\left(\widehat{x},\frac{\nu_2}{\nu_1}\right)}{u'\left(\varphi\left(\widehat{x},\nu_2\right)\right)\varphi_\nu\left(\widehat{x},\nu_2\right)} \geq 1$$

For HARA utility functions $u(c) = \xi\left(\eta + \frac{c}{\gamma}\right)^{1-\gamma}$ this writes:

$$\begin{aligned}(1+t_1)^{2-\gamma}t_2^{1+\gamma} + (1+t_2)^{2-\gamma}t_1^{1+\gamma} &\geq (t_1+t_2)^{2-\gamma} \\ \text{with } t_i &= \nu_i^{1/\gamma}\end{aligned}$$

which is always true for ν_1 and ν_2 positive when $\gamma \geq \frac{1}{2}$ (this can be prooved by showing that, when $\gamma \geq \frac{1}{2}$, the minimum value of the left hand side is higher than the right hand side)

We conclude then that when $\gamma \geq \frac{1}{2}$, $\frac{\partial\Omega}{\partial\theta}$ is negative. Therefore, in the plane (ν_1, ν_2) the curves $\pi(\underline{\theta}) \bullet \delta = 0$, $\pi(\overline{\theta}) \bullet \delta = 0$ cross once at $(\nu_1, \nu_2) = (1, 1)$.

Lastly, we prove that, at the optimum, we have necessarily $\nu_2 > 1$.

Under the optimal contract,

$$\underline{v} \equiv \begin{pmatrix} 0 \\ u\left(\varphi\left(\widehat{x}, \frac{\nu_1}{\nu_2}\right)\right) - u(\widehat{x}) + u(\widehat{x}) - u(\underline{x}) \\ u\left(\varphi\left(\widehat{x}, \frac{\nu_2}{\nu_1}\right)\right) - u(\widehat{x}) + u(\widehat{x}) - u(\overline{x}) \\ 0 \\ u\left(\varphi(\overline{x}, 1/\nu_1)\right) - u(\overline{x}) \\ u\left((\widehat{x}, 1/\nu_2)\right) - u(\widehat{x}) + u(\widehat{x}) - u(\underline{x}) \\ u\left(\varphi(\widehat{x}, 1/\nu_1)\right) - u(\widehat{x}) + u(\widehat{x}) - u(\overline{x}) \\ u\left(\varphi(\underline{x}, 1/\nu_2)\right) - u(\underline{x}) \end{pmatrix},$$

and we can easily prove by contradiction that when $\dfrac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} < \dfrac{(1-\underline{\theta})\mathrm{E}(\theta)}{\underline{\theta}(1-\mathrm{E}(\theta))}$,
$\pi(\underline{\theta}) \bullet \underline{v} \geq 0 \Rightarrow \nu_2 > 1$

2.7 Appendix B: Extension to the case of correlated risk types

The model presented in our paper can be generalized to correlated risk types. As in usual Bayesian implementation this however adds technical difficulties. It is still worthwhile to study this extension in the case of mutual agreements.

Correlated types corresponds in our setting to situations where an exogenous variable impacts the risk exposure of both individuals. In the case of car insurance for example, the state of the road and the traffic may influence the probability of accident of each individual, but realizations of the risk remain independent.

In such a configuration, we prove in this section that most of the propositions (namely propositions 2.1 to 2.3) of the paper are generalizable. We first show that equal sharing of wealth is achievable if risk aversion is high, heterogeneity low or if a low risk agent has a high probability to be matched with another low risk individual. Under complete information, the mutuality principle continues to hold even when previous conditions are not met. This is not the case as soon as risk types become private information. Then, as in the case of independent risk types, the asymmetry of information makes ex-post equal sharing unsustainable when both individuals are low risk types and induces some exchanges when agents have the same level of initial wealth.

The introduction of correlated types however worsen the issues of showing that only one constraint binds at the optimum. In the present section, we are only able to describe the optimum assuming that only one incentive constraint binds. Under this assumption, the only possible configuration under which the direction of transfers is not changed with the introduction of asymmetric information is negative correlation of types.

Still, we emphasize in this extension the role of conditional distribution of risk types on efficiency. When the conditional probability for a low risk type agent to be paired with an other low risk type individual ($\text{prob}(\theta_2 = \underline{\theta}|\theta_1 = \underline{\theta})$) is high, there is no loss of efficiency due to the asymmetry of information. However, when this conditional probability is low, the asymmetry of information leads to a loss of efficiency entirely bore by low risk agents. Moreover, this effect on efficiency is all the more important that the conditional probability for a high risk agent to be paired with an other risky individual $\big(\text{prob}(\theta_2 = \overline{\theta}|\theta_1 = \overline{\theta})\big)$ is high.

To go further in the analysis of correlated type let us define the ex ante distribution of types – that is assumed to be common knowledge – by:

$\overline{\overline{\rho}} \equiv \rho(\overline{\theta}, \overline{\theta}) \equiv \text{prob}(\theta_1 = \theta_2 = \overline{\theta})$

$\underline{\overline{\rho}} \equiv \rho(\overline{\theta}, \underline{\theta}) \equiv \text{prob}(\theta_1 = \overline{\theta}, \theta_2 = \underline{\theta}) \equiv \text{prob}(\theta_1 = \underline{\theta}, \theta_2 = \overline{\theta}) \equiv \rho(\underline{\theta}, \overline{\theta})$

$\underline{\underline{\rho}} \equiv \rho(\underline{\theta}, \underline{\theta}) \equiv \text{prob}(\theta_1 = \theta_2 = \underline{\theta})$.

Assuming first complete information on types (after the design of the agreement), the program of the principal now writes

$$\max_x \quad \sum_{\Theta^2} \rho(\theta_1, \theta_2) \sum_{\Omega} \pi(\theta_1, \theta_2, \omega) \left[u(x_1(\theta_1, \theta_2, \omega)) + u(x_2(\theta_1, \theta_2, \omega))\right] \tag{2.25}$$

$$\text{s.t.} \quad \begin{cases} x_1(\theta_1, \theta_2, \omega) + x_2(\theta_1, \theta_2, \omega) = X(\omega) & \forall \theta_1, \theta_2, \omega \\ \sum\limits_{\theta_2 \in \Theta} \rho(\theta_1, \theta_2) \sum\limits_{\Omega} \pi(\theta_1, \theta_2, \omega) \left[u(x_1(\theta_1, \theta_2, \omega)) - u(X_1(\omega))\right] \geq 0 & \forall \theta_1 \\ \sum\limits_{\theta_1 \in \Theta} \rho(\theta_1, \theta_2) \sum\limits_{\Omega} \pi(\theta_1, \theta_2, \omega) \left[u(x_2(\theta_1, \theta_2, \omega)) - u(X_2(\omega))\right] \geq 0 & \forall \theta_2 \end{cases}$$

Therefore, the equal sharing rule satisfies the participation constraints if

$$\frac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} \geq \frac{(1 - \theta_1) E(\theta_2|\theta_1)}{\theta_1 (1 - E(\theta_2|\theta_1))} \ \forall \theta_1$$

Remark 2.1 *Risk types are positively correlated if* $\widehat{\theta} \equiv E(\theta_2|\theta_1 = \overline{\theta}) \geq E(\theta_2|\theta_1 = \underline{\theta}) \equiv \widetilde{\theta}$, *negatively correlated if* $\widehat{\theta} \leq \widetilde{\theta}$, *and independent if* $\widehat{\theta} = \widetilde{\theta}$.

As $\widehat{\theta} \leq \overline{\theta}$ and $\underline{\theta} \leq \widetilde{\theta}$, $\dfrac{(1-\overline{\theta})\widehat{\theta}}{\overline{\theta}(1-\widehat{\theta})} \leq 1 \leq \dfrac{(1-\underline{\theta})\widetilde{\theta}}{\underline{\theta}(1-\widetilde{\theta})}$ and the IPC is always verified for $\theta_1 = \overline{\theta}$. $\left(\frac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})} \geq 1\right)$. Therefore, the equal sharing rule satisfies the interim participation constraints if :

$$\frac{u(\widehat{x}) - u(\underline{x})}{u(\overline{x}) - u(\widehat{x})} \geq \frac{(1-\underline{\theta})\widetilde{\theta}}{\underline{\theta}(1-\widetilde{\theta})}$$

Noticing that the right hand side can be written as $1 + \dfrac{\underline{\overline{\rho}}}{\underline{\underline{\rho}} + \overline{\overline{\rho}}} \dfrac{\overline{\theta} - \underline{\theta}}{\underline{\theta}(1-\widetilde{\theta})}$, it follows that equal sharing is more easily to sustain when the probability for a low type risk to be matched with a risky agent $\left(\text{prob}(\theta_2 = \underline{\theta}|\theta_1 = \overline{\theta})\right)$ is low, when heterogeneity $\overline{\theta} - \underline{\theta}$ is high (the right hand side increases with a mean-preserving spread of the probability of damage) or when risk aversion is sufficiently high.

Using the same kind of proof than in the independent case, we moreover can show that when equal sharing in not sustainable, the optimal agreement exhibits the same properties as in Proposition 1. The generalized proposition becomes:

Proposition 1g *When individual risk types are public information, the optimal risk sharing rule* $x_1((\theta_1, \theta_2, \omega))$, $x_2(\theta_1, \theta_2, \omega)$*:*

(i) *always satisfies the mutuality principle*

(ii) *corresponds to equal sharing of wealth in any configuration if risk aversion is high, heterogeneity is low or when a low risk agent is very likely to be matched with another low risk type*

(iii) *if risk aversion is too low, heterogeneity too high or when a low risk indivifual has a high probability to face a risky agent*

  (a) *equal sharing is optimal when agents share the same risk type*

  (b) *a low risk type agent always gets more than average wealth when matched with a high risk*

Under asymmetry of information, when risk type are correlated, the Bayesian incentive constraints write

$$\sum_{\theta_2\in\Theta}\rho(\overline{\theta},\theta_2)\sum_{\Omega}\pi(\overline{\theta},\theta_2,\omega)\left(u(x_1(\overline{\theta},\theta_2,\omega))-u(x_1(\underline{\theta},\theta_2,\omega))\right)\geq 0$$
$$\sum_{\theta_2\in\Theta}\rho(\underline{\theta},\theta_2)\sum_{\Omega}\pi(\underline{\theta},\theta_2,\omega)\left(u(x_1(\underline{\theta},\theta_2,\omega))-u(x_1(\overline{\theta},\theta_2,\omega))\right)\geq 0$$
$$\sum_{\theta_1\in\Theta}\rho(\theta_1,\overline{\theta})\sum_{\Omega}\pi(\theta_1,\overline{\theta},\omega)\left(u(x_2(\theta_1,\overline{\theta},\omega))-u(x_2(\theta_1,\underline{\theta},\omega))\right)\geq 0$$
$$\sum_{\theta_1\in\Theta}\rho(\theta_1,\underline{\theta})\sum_{\Omega}\pi(\theta_1,\underline{\theta},\omega)\left(u(x_2(\theta_1,\underline{\theta},\omega))-u(x_2(\theta_1,\overline{\theta},\omega))\right)\geq 0$$

As in the independent case, the equal sharing rule satisfies all the above constraints. Therefore Proposition 2 also generalizes and becomes

Proposition 2g *When risk is private information, equal-sharing rule is optimal when* $\dfrac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})}\geq\dfrac{(1-\underline{\theta})\widetilde{\theta}}{\underline{\theta}(1-\widetilde{\theta})}$.

We can also prove, applying to correlated type the same demonstration as in the independent case, that Proposition 3 also holds in the present case.

Proposition 3g *When risk types are private information, and equal sharing is not optimal, there exists a unique optimal welfare improve contract, such that*

(i) The mutuality principle is violated;

(ii) Ex-post equal sharing is not achievable when both agents turn out to be low risk type

(iii) The optimal agreement implies some exchanges when agents have the same initial wealth level.

Once again, this properties are mainly due to the distortion of $x_i(\underline{\theta},\underline{\theta},\omega)$ necessary to prevent high risk from cheating. Moreover, we can show using, as previously, proofs by contradiction that the participation constraint of low risk and the incentive constraint of high risk individuals necessarily bind at the optimum, whereas the participation constraint of high risk agents never binds. However, we are unable to generalize here the proof showing that the incentive constraint of less risky agents is lax at the optimal. We are thus forced to assume that only one incentive constraint binds at the optimum to generalize Proposition 4.

Proposition 4g *Assuming that only one incentive constraint binds at the optimum, when* $\frac{u(\widehat{x})-u(\underline{x})}{u(\overline{x})-u(\widehat{x})} < \frac{(1-\underline{\theta})\widetilde{\theta}}{\underline{\theta}(1-\widetilde{\theta})}$

(i) *ex-post equal sharing is*

- *optimal when both agents are risky individuals:* $x_1(\overline{\theta},\overline{\theta},\omega) = X(\omega)/2\ \forall\omega$
- *not incentive compatible if they are of low risk type:* $x_1(\underline{\theta},\underline{\theta},(0,1)) > \widehat{x}$.

(ii) *When agents announce different types*

- *the direction of the transfer when both agents suffer the damage changes relative to the complete information benchmark if risk types are independent or positively correlated:* $\nu_{g1} < 1$ *and* $\nu_{g2} > 1$ *with*

$$\nu_{g1} \equiv \frac{u'(x_2(\overline{\theta},\underline{\theta},(0,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,0)))} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,0)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,0)))} < \frac{u'(x_2(\overline{\theta},\underline{\theta},(1,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(1,1)))} = \frac{u'(x_2(\overline{\theta},\underline{\theta},(0,1)))}{u'(x_1(\overline{\theta},\underline{\theta},(0,1)))} \equiv \nu_{g2}$$

(iii) *If risk are negatively correlated, the low risk agent may still optimally get more than average wealth in every stats (as in the complete information benchmark):* $\nu_{g1},\ \nu_{g2} < 1$ *if* $\frac{1+\underline{\gamma}-\frac{\overline{\overline{\rho}}.\overline{\theta}}{\underline{\rho}.\underline{\theta}}\overline{\lambda}}{1+\overline{\lambda}} \geq 1$ *;* $\nu_{g1} < 1$ *and* $\nu_{g2} > 1$*, otherwise* *where* $\underline{\gamma}$ *and* $\overline{\rho}$ *represent respectively the Lagrange multipliers assigned to the participation constraint of low type and the Bayesian incentive constraint of high risk type agents.*

Oppositely to the case of independent risk types, it then appears that the mechanism on $x_i(\underline{\theta},\underline{\theta},\omega)$ may be sufficient to prevent high risk type from cheating when risk types are negatively correlated. Indeed, this distortion mainly prevents from cheating when high risk agents have a high probability to be confronted to a low risk agents.

We have shown previously that this mechanism is not sufficient when risk is independent as then, risky individual also need to be prevented from cheating when matched with another high risk agent. In this case, the optimal mechanism necessarily specifies change in the direction of some transfers relatively to complete information. Proposition 4g first generalizes this property to positively correlated types. This seems pretty intuitive as then the probability for a high type agent to be confronted to another risky individual $\big(\text{prob}(\theta_2 = \overline{\theta}|\theta_1 = \overline{\theta})\big)$ is large. However, when this conditional probability is low, there is no need to change the direction of transfer when agents announce different types to be incentive compatible. Proposition 4g indicates that this can only happens when risk is negatively correlated. In some cases, the distortion of the first best when both agents announce low risk type is then sufficient to prevent high risk agents to cheat on their type.

Assuming that only one incentive constraint bind at the equilibrium, the effect of the asymmetry of information on efficiency thus highly depends on the conditional distribution of risk type. When the conditional probability for a low risk type agent to be paired with an other low risk type individual $(\text{prob}(\theta_2 = \underline{\theta}|\theta_1 = \underline{\theta}))$ is high, there is no loss of efficiency due to the asymmetry of information. In this sense, mutual agreements seem to be better adapted to asymmetric information than insurance companies. However, when this conditional probability is low, the asymmetry of information leads to a loss of efficiency. As in Rothschild and Stiglitz [62] this loss is entirely bore by low risk agents. Moreover, this effect on efficiency is all the more important that the conditional probability for a high risk agent to be paired with an other risky individual $\big(\text{prob}(\theta_2 = \overline{\theta}|\theta_1 = \overline{\theta})\big)$ is high.

On the basis of various simulations with logarithmic and C.A.R.A. (Constant Absolute Risk Aversion) utility functions it furthermore turns out that, depending on risk type correlation, the properties of the optimal contract when agents announce different type can be summarized in the following picture

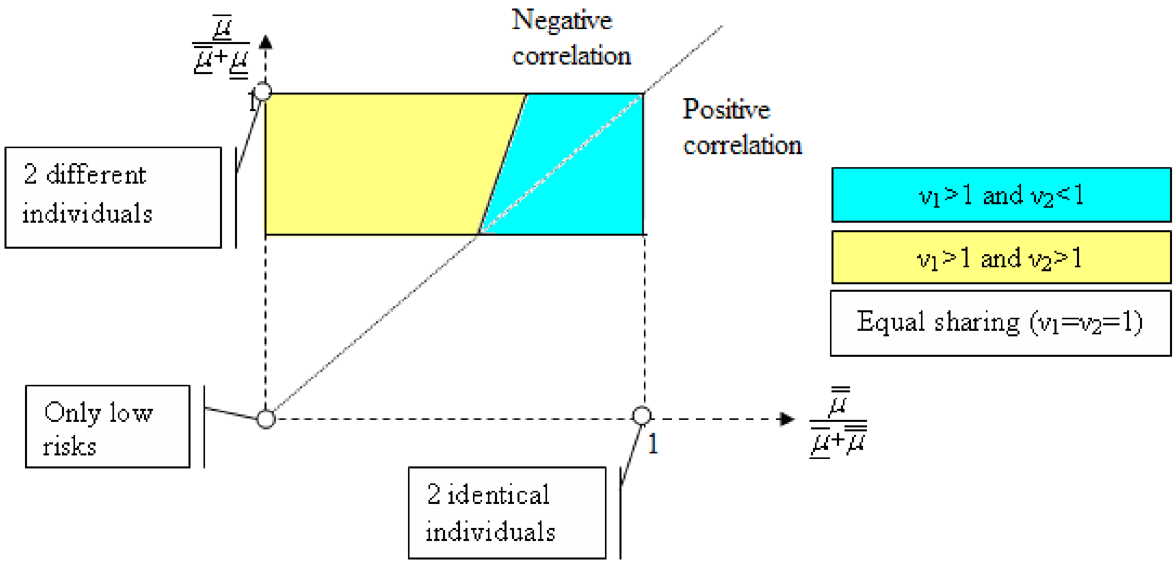


Figure 2.1: The Optimal Contract Under Asymmetric Information

For each value of conditional probability prob $\left(\theta_2 = \overline{\theta} | \theta_1 = \underline{\theta}\right)$ non compatible with equal sharing, it seems to exist a maximal conditional probability prob $\left(\theta_2 = \overline{\theta} | \theta_1 = \overline{\theta}\right)$ $\Big($ with prob$\left(\theta_2 = \overline{\theta} | \ \theta_1 = \underline{\theta}\right) >$ prob$\left(\theta_2 = \overline{\theta} | \theta_1 = \overline{\theta}\right)\Big)$ for which the asymmetry of information does not change direction of transfers. It is moreover meaningful to note that in every performed simulation, the Bayesian incentive constraint of low risk type agents never binds at the optimum.

Most of our paper thus appears to be generalizable to the case of correlated risk type. First, equal sharing of wealth is achievable if risk aversion is high, heterogeneity low and if a low risk agent has a high probability to be matched with another low risk individual.

Moreover, the mutuality principle always holds under complete information, but fails to hold when risk types are private information and equal sharing is not sustainable. Then, the asymmetry of information makes ex-post equal sharing unsustainable when both individuals are low risk types and induces some exchanges when agents have the same level of initial wealth. However, the introduction of correlation in risk types worsen our technical difficulties. Still, assuming that only one incentive constraint binds at the equilibrium, we show that the loss of efficiency due to asymmetric information highly depends on conditional risk distribution. When risk types are negatively correlated, the optimal contact may moreover (oppositely to the independent case) give more than average wealth to a low risk agent when matched with a more risky agent and still be incentive compatible.

Chapter 3

Moral hazard in dynamic insurance, Classification Risk and Prepayment

3.1 Introduction

Thanks to technological progress in medicine, people tend to live longer. If most individuals also live healthier[1], these technological progress moreover allows patients affected by chronic illness to have higher life expectancy. This raises a new issue in medical insurance, as insurers tend to charge this new class of agents – that needs to be covered for a long time against high expected heath cost of treatment – with high premia. It therefore creates – for agents that contract a chronic condition – a risk of being reclassified "high-risk" by his insurer and therefore to pay a high premium. Insurance literature refers to this risk as the classification risk.

One possible option for reducing these premia is the use of to dynamic insurance, that is of long-term insurance contract. This allows for risks mutualization through intra- and inter-generational insurance. On the one hand, younger agents can subsidize older ones, as they expect to benefit form similar subsidies when old. This subsidy corresponds to early payment of future premia, and is referred to as front-loading.

[1]The literature on medical science refers to this phenomenon as healthy ageing

On the other hand, when old, low risk may subsidize high risk agents. These two mechanisms tend to decrease the premium of old agents highly exposed to risk, and therefore reduce classification risk.

This process however introduces new issues. First, intra-generational insurance may lead to the exit of low risk policyholders. Insurance contract are characterized by one-side commitment and agents that turn out to be low risk may have an incentive to leave the contract if they find more profitable outside options (spot market for example). This phenomenon is sometimes refereed to as (cream)-skimming.

The reduction of classification risk moreover raises a moral hazard issue. Being insured against the risk of being considered high risk reduces the incentive to exert preventive efforts that decreases the probability of becoming more risky. In the following we will refer to such effort - that can correspond to safe behavior or to "hygieno-dietetic" regime for example - as primary prevention [2].

Recent initiatives on insurance market have highlighted the interest for insurers, and especially mutual insurers, for primary prevention. In 2005 and 2007 respectively, French mutual insurers AGF and MAAF have begun to reimburse some alimentary products designed to lower the cholesterol level. A similar program has also been introduced in 2005 by the Dutch insurer VGZ.

[2] This terminology comes from medical science. The U.S. Preventative Services Task Forces' Guide to Clinical Preventive Services (2d edition, 1996) defines primary prevention as interventions that reduce the risk of disease occurrence in otherwise healthy individuals. Secondary prevention measures then corresponds to identifying and treating "persons who have already developed risk factors or preclinical disease but in whom the condition is not clinically apparent". It thus can be related to "self-protection" (see Ehrlich and Becker [23]) or "loss prevention" (see Mehr and Commack [51]). Finally, tertiary prevention concerns "care of established disease, with attempts made to minimize the negative effects of disease" and thus correspond to "self-insurance" in Ehrlich and Becker [23] or "loss protection" in [51].

The aim of the present paper is to introduce the notion of primary prevention in dynamic insurance. We define the optimal dynamic contract under moral hazard on the probability of becoming high risk. We then analyze the impact of moral hazard on front-loading, classification risk and cream-skimming. This way we are able to infer the stability of dynamic insurance contract that account for preventive effort.

To do so we build a two-period model of mutual health insurance. During the first period, agents are identically exposed to health risk and can invest in primary prevention. In period 2, agents can either be high risk or low risk type. The amount of effort spent in period 1 reduces the probability of being high risk that is the probability of having a high probability of falling ill in period 2. When effort is observable and contractible upon, dynamic insurance fully insures against classification risk. However, when effort is unobservable, the insurance offered during the second period depends on risk type (that we assume to be observable and public information). This raises the issue of classification risk.

This chapter highlights a trade-off between two behaviors toward this future risk. On the one hand, thanks to dynamic insurance, agents can transfer wealth between the two periods through prepayment of premia (i.e. intergenerational insurance). By paying a higher premium during the first period, they can reduce second period premia and classification risk. Such a mechanism can therefore be related to precautionary savings[3] and to the notion of prudence. On the other hand, to reduce the classification risk, agents can exert effort of primary prevention. Such an effort, that reduces the probability of being high risk in second period, seems to be related the concept of risk aversion (see. Jullien et al. [40] for a discussion on the link between effort of prevention and risk aversion). This suggests that the trade-off between the two means of reducing classification risk depends on the ratio of (absolute) prudence to (absolute) risk aversion.

[3] By allowing an unequal repartition of the prepaid premium between the two states of the second period, prepayment appears however more flexible than precautionary saving

The present paper confirms this intuition and shows that the critical level of this ratio is 2. If absolute prudence is larger than twice absolute risk aversion, to respond to future uncertainty, the agents transfers wealth in second period rather than exerts effort, that is they foresee rather than prevent. We will then refer to these agents as "farsighted". We show in this paper that, in case of farsighted agents, moral hazard (through the unobservability of preventive efforts) increases the first period premium (and hence enhances prepayment). On the contrary, if agents are not farsighted, that is if they favor prevention (rather than inter-period transfers), moral hazard reduces classification risk and intergenerational insurance. As a scope for public action, we also prove that the classification risk can be reduced by a decrease in the cost of prevention (whatever the degree of foresight) or by increasing the effectiveness of prevention when agents are farsighted.

With CRRA (Constant Relative Risk Aversion) preferences, it appears that an increase in agents' degree of foresight leads to a decrease in the premium offered to low risk agents in second period, if the cost of effort is low enough. Then, the more farsighted its policyholders, the less contestable (by spot insurers) a mutual insurer that offers long-term contracts. After having defined a suitable utility function - that satisfies the simplifying property of having a linear reciprocal derivative - we moreover show that the various degrees of front-loading and lapsation observed in insurance contracts (see Hendel and Lizzeri [36]) can be explained by heterogeneity in agents' preferences. Lastly, we highlight the fact that cross-subsidization allows to increase front-loading for agents with a strong preference for present and therefore to improve the stability of dynamic contracts.

The issue of classification risk has received significant attention in the literature on long term insurance. To reduce this risk that may make insurance unaffordable for most risky agents, Pauly et al. [58] propose guaranteed renewable insurance policies, that consists in a declining schedule of premia over time. They construct a scheme in which the premium is always lower that the expected future lifetime expenses of the lowest risk buyers. Frick [27] however points out that such solution may not be observed as it involves high premia early in life. If agents are too impatient and if they face borrowing constraint, they will at most purchase partially guaranteed renewable insurance.

Alternatively, Cochrane [14] proposes time-consistent insurance contracts that provide insurance against classification risk using severance payments. When an agent turn out to be high risk, she receives a lump sum equals to the increased present value of his premium. Severance payments compensate for changes in premium and allow every consumer to purchase insurance at his actuarially fair premium. Pauly et al. [59] argue that the effectiveness of this scheme highly relies on the assumption of perfect credit market, and Hendel and Lizzeri [36] point out that such contracts can not be implemented in life insurance for legal reasons.

In a two-period model similar to ours, Hendel and Lizzeri [36] analyzes to what extend prepayment of premia (front-loading) can reduce classification risk when accounting for cream-skimming. They state that front-loading allows reducing both cream-skimming (low risk are insured at their fair premium in second period) and classification risk (agents with different types have the same insurance contract)[4]. On the basis of these findings, they moreover argue that the various degrees of front-loading and lapsation observed in insurance data can be explained by heterogeneous costs of front-loading (that is by heterogeneous profiles of income growth).

[4]Contrary to us, Hendel and Lizzeri [36] allow for more than two risk types in second period

Through primary preventive effort we add moral hazard to this model. To be incentive compatible, the optimal contract then necessarily specifies different insurance schemes to different types. This offers an alternative explanation to the stylized fact highlighted by Hendel and Lizzeri [36]. Accounting for moral hazard, the observed variety of front-loading and lapsation, can be explained by heterogeneous behavior toward risk (risk aversion and prudence).

Our paper is not the first attempt to introduce moral hazard in dynamic insurance contract. Abbring et al [1] use dynamic insurance contract in their empirical study on the distinction between moral hazard and adverse selection. They however analyze moral hazard on the probability of accident, that is secondary prevention. We focus here on primary prevention. An important implication of the difference is that primary prevention - contrary to secondary prevention - leads to dynamic moral hazard. The effort exerted in current period reduces the probability of being high risk next period.

Nishimura [56] analyzes the effect of primary prevention on front loading in Hendel and Lizzeri's life insurance model. He characterizes the condition under which the optimal contract involves front-loading. The emergence of front-loading depends on agents' risk aversion and on the effectiveness of prevention. He then studies capital market structure and the scope for government action. We however want to focus here on classification risk and on the conditions under which dynamic contracts can prevent from agents to leave the company when they turn out to be low risk.

In the next session, we define the notion of foresight and analyze in which context it arises in the economic literature. We present the model in section 3. The optimal dynamic contract under moral hazard is defined in section 4 and general results of comparative statics are provided in Section 5. To go further in the analysis of comparative foresight we then rely on specific utility function in section 6, and we interpret our results in term of sustainability of dynamic insurance with respect to spot insurance in section 7.

A brief extension with cross-subsidization among agents heterogeneous in foresight is provided in section 8. We present a possible application of our model to unemployment and life insurance in Section 9, and eventually outline our conclusion and directions for future research in Section 9.

3.2 The notion of foresight

It is now well established that the inverse of marginal utility $(1/u')$ plays a preponderant role in models with model hazard. Our paper emphasizes the influence of the degree of concavity of this function, that is $(1/u')''$. This is not the first work where the second derivative of the inverse of marginal utility matters. In a principal-agent model with moral hazard, Newman [55] shows that, if $1/u'$ is convex, an increase in wealth makes the incentive scheme more expensive for the principal. Then, the higher the initial wealth of the agent, the lower the expected profit of the principal. Thiele and Wambach [67] generalize this condition and state that for this statement to hold, it is sufficient to assume that an agent with utility function $1/u'$ is less risk adverse that an agent with utility function u. This last condition corresponds to an absolute index of prudence (introduced by Kimball [42]) smaller than three times the absolute index of risk-aversion ($P \leq 3A$) when the convexity of the inverse of marginal utility writes $P \leq 2A$ [5]. In the general case, Amir and Czupryna [2] prove that $P \geq kA$ is equivalent to (k-2)-concavity of the first derivative of the inverse utility function. Moreover, Eeckhouldt and Gollier [22] show that two independent risks are substitutes if absolute prudence is decreasing and larger than twice the absolute risk aversion.

[5] With constant relative risk aversion utility functions these condition is equivalent to a degree of relative risk aversion larger than 1/2 and 1 respectively

The second derivative of the inverse of marginal utility function is also crucial in models with uncertainty in the probability of damage. In studying environmental problems and the 'precautionary principle', Gollier et al. [32] build a two-period model where the uncertain damage in second stage depends on the consumption of both periods. They moreover assume that researchers work on (Bayesianly) revising the beliefs on the distribution of uncertainty. One of their main results is that a better information structure (that is beliefs more dispersed) decreases (resp. increases) the efficient level of consumption in first stage if $P \geq 2A$ (resp. $P \leq 2A$). In this sense, if prudence is larger than twice risk aversion, progress induces precautionary savings.

Gollier [31] finds a similar result when analyzing a dynamic model of prevention with uncertainty on the probability of loss and bayesian revisions. He assumes that, in both periods, agents can exert an effort of (tertiary) prevention that reduces the amount of loss in case of damage and have the possibility to save during the first period. Moreover, agents revise their belief about the probability of loss in the second period on the basis of what they have observed during first stage. His main results are that (i) an increase in the expected probability of loss increases (resp. decrease) the marginal value of savings if $1/u'$ is convex (resp. concave) and (ii) the uncertainty on the probability of loss increase (resp. decrease) the efficient level of first stage effort if $P \leq 2A$ (resp. $P \geq 2A$). Our model differs from Gollier's in at least two respects. First of all we allow for insurance in both stages when Gollier [31] only models savings in first stage. In the present paper, risk (of classification) is then endogenous as it depends on insurance offered in both states of nature in second period. Moreover, whereas Gollier studies self protection and bayesian revision of probability we analyze in this paper the effort of primary prevention that is an effort that impacts the probability of having a high probability of damage.

This brief review of literature highlights the trade-off that arises from an increase in future uncertainty, between a decrease in present consumption (that is an increase in savings) and an increase in effort. If $P \geq 2A$, that is if $1/u'$ is concave, the precautionary motive dominates and to face uncertainty, the agents save rather than exert the effort. We therefore define such agents, that foresee rather than prevent, as being "farsighted".

Definition 3.1 *In the following, an agent will be said to be "farsighted" if and only if the inverse of the marginal utility is concave, that is if the index of absolute prudence is larger that twice the index of risk aversion.*

3.3 The Model

To analyze the impact of moral hazard on prepayment and classification risk, we build an overlapping generation model (to capture the case of social insurance) with change in risk exposure during the life cycle. We model the simplest 2-period, 2-type case and assume that (homogeneous) newborn agents can affect their second period health status through primary prevention.

Consider overlapping generations (of same size) living for two periods $t = 1, 2$. At each period, identical agents receive a sure revenue R. During the first period, (young) individuals face the same risk, that is the same probability q_1 of suffering a loss L. Let us note $K_1 \equiv -q_1 L$ the expected loss, that is the expected health cost in the case of health insurance. At $t = 2$, (old) agents may be of two types. Either, with probability p, they are low risk type and face a probability of loss q_2^l ($K_2^l \equiv q_2^l L$) or, with probability $1 - p$, they are high risk type and suffer loss with probability q_2^h, with $q_2^l < q_2^h$ (therefore $K_2^l < K_2^h \equiv q_2^h L$).

Information about agents' risk type is revealed at the beginning of second period (for example through medical check-ups) and is then public information. Young agents can exert a primary preventive effort that reduces the probability of becoming high risk type in second period [6]. We assume that agents choose among two levels of prevention $\underline{e}$ and $\overline{e}$ ($\underline{e} < \overline{e}$) leading respectively to probabilities of being low risk $p(\underline{e}) \equiv \underline{p}$ and $p(\overline{e}) \equiv \overline{p}$, with $\underline{p} < \overline{p}$. Let us note $\Delta p \equiv \overline{p} - \underline{p}$.

Let X_i^j be the wealth of agents of type j in period i. In the absence of insurance, the income profile of a newborn agent can be schematized as follows

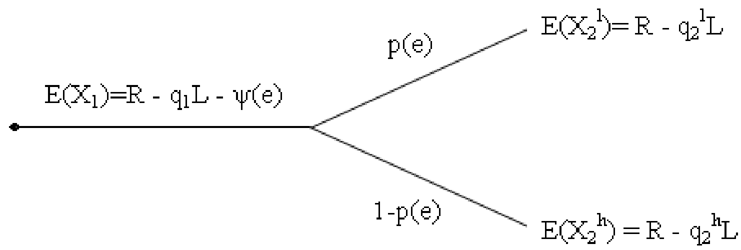


Figure 3.1: The income profile without insurance

During the first period, the utility function is supposed to be separable in wealth and effort and the utility-cost of exerting high effort of primary prevention is noted $\psi \equiv \psi(\overline{e}) - \psi(\underline{e})$. We moreover assume time separability of preference (to distinguish saving and insurance behavior)[7], and for the sake of simplicity, that utility is linear in wealth during the first period.

Let us note $u(.)$ (with $u'(.) > 0$ and $u''(.) < 0$) the utility function of both types of old agents. We assume that there is no direct utility loss due to health status. This would anyway worsen the welfare of high risk agents and therefore advocate for a lower classification risk.

[6]In our setting, an effort of secondary prevention would reduce q_2^l and q_2^h, and corresponds to self-protection; whereas effort of tertiary prevention reduces L and corresponds to self-insurance

[7]The use of incentive constraints prevent us to model Kreps-Porteus preferences. This would allow us to fully disentangle risk and time effects. However, the use of such non-expected utility makes the problem untractable as it greatly complicates the writing of the incentive compatible constraint

To be insured against this two-period risk, a (benevolent) mutual insurer offers to young agents a dynamic insurance contract, that is a contract specifying premia and coverage for both periods and depending on risk status in second period[8]. The timing of the game is described in Figure 3.2

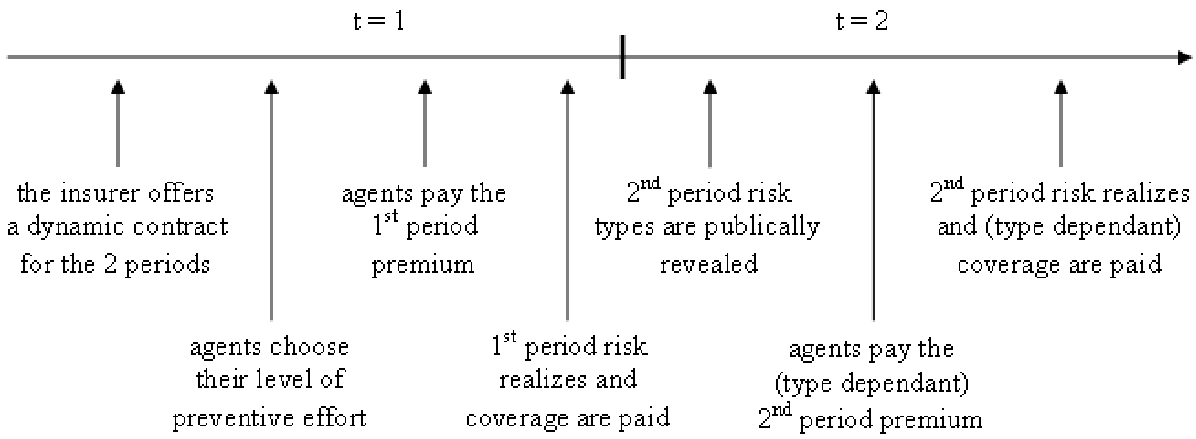


Figure 3.2: The timing of the game

We moreover assume that there is no market insuring against the classification risk as proposed by Tabarrok [66] when studying the issue of genetic testing (he calls it "genetic insurance"). Here, dynamic insurance contracts allow the insurer to use prepayment of premia in first period to decrease the premium offered to high risk type agents when old (as shown in Hendel Lizzeri [36]). However, when effort is unobservable and not contractible upon, this may lower the incentive to exert high preventive effort. The aim of this paper is thus to analyze the trade-off resulting from a decrease in premium of high risk in second period, between an increase in insurance and a decrease in the incentive for primary preventive effort. This then allows us to analyze under which condition the optimal contract resulting from this trade-off is sustainable when we introduce the possibility of one-period (spot) insurance.

[8]The reader should note that the following problem also fits in the case of competing insurance companies that do not seek to propose profitable one-period contract. We briefly discuss this alternative interpretation in Section 6.

Note here that prepayment of premia can be related to precautionary saving as it corresponds to an intertemporal transfer of wealth used to deal with future uncertainty. These two mechanisms are however different in fundamental respect that plays a important role in our setting. Indeed, whereas savings have the same return in every future state (and therefore don't have much impact on classification risk), the insurance company can choose to reallocate the prepaid part of premia differently across the states (what can have an important impact on classification risk).

3.4 The optimal dynamic contract

3.4.1 The benchmark case of observable effort

It is first easy to show, using the concavity of the utility function, that the dynamic insurance contract necessarily specify complete insurance (in the sense that it provides an agent with the same wealth whether she suffers the damage or not) once risk types are known [9]. A dynamic contract is therefore fully defined by a triplet $(\Pi_1, \Pi_2^l, \Pi_2^h)$ of premia corresponding respectively to the expected costs K_1, K_2^l and K_2^h, and the coverage is in any cases equal to the amount of the loss L. Under this dynamic insurance contract, the income profile of a newborn agent can then be schematized as follows:

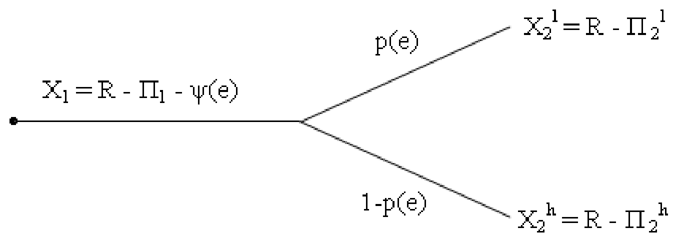


Figure 3.3: The income profile under the insurance contract

[9] This issue is more problematic in Hendel and Lizzeri [36] and Nishimura [56] that model life insurance and therefore specify state (alive/dead) dependent utility function

The risk of being classified high risk is then measured by the difference between the second period premia.

Definition 3.2 *The classification risk corresponds to the risk of being classified high risk by his insurer and therefore to paid a high premium. In our two-type model with complete insurance in each state, this risk is simply measured by the spread between the premia paid by each type in second period:* $\Pi_2^h - \Pi_2^l$

The mutual insurance firm, as non profit organization, then seek to maximize the expected utility of a young individual that exert an effort e:

$$(R - \Pi_1) - \psi(e) + p(e)u\left(R - \Pi_2^l\right) + (1 - p(e))u\left(R - \Pi_2^h\right) \tag{3.1}$$

By definition the use of external capital is excluded in a mutual organization. However, if the mutual insurer is large enough, it can rely on the law of large numbers, and the zero profit condition writes

$$\Pi_1 + p(e)\Pi_2^l + (1 - p(e))\Pi_2^h = K_1 + p(e)K_2^l + (1 - p(e))K_2^h \equiv E(K|e) \tag{3.2}$$

This states that the sum of premia collected (from young and old agents) allows (in expectation) for the reimbursement of heath costs. Recall here that we consider an overlapping generation model with identical agents (that therefore exert the same prevention effort) and generations of the same size.

With observable effort it is then optimal to set ${\Pi_2^l}^* = {\Pi_2^h}^* = \Pi_2^*$ such that $u'(R - \Pi_2^*) = 1$ and $\Pi_1^* = E(K|e) - \Pi_2^*$. Therefore, the optimal premia in second stage are independent of the level of preventive effort and there is no classification risk at the optimum. However, the premium paid at first stage is decreasing with the level of effort as $p(\overline{e}) > p(\underline{e}) \Rightarrow \overline{K} \equiv E(K|e = \overline{e}) < E(K|e = \underline{e}) \equiv \underline{K}$.

Therefore, without moral hazard, assuming $\underline{K} - \overline{K} > \psi$ the optimal contract specifies:

$$\begin{cases} e = \overline{e} \\ {\Pi_2^l}^* = {\Pi_2^h}^* \equiv \Pi_2^* \text{ with } u'(R - \Pi_2^*) = 1 \\ \Pi_1^* = \overline{K} - \Pi_2^* \end{cases} \tag{3.3}$$

3.4.2 The optimal dynamic contract under moral hazard

Now, if efforts of primary prevention are not observable, that is under moral hazard, agents have an incentive the exert the maximal level of effort if the insurance contract satisfies

$$u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \frac{\psi}{\Delta\pi} \tag{3.4}$$

Therefore, the optimal contract that gives the incentive to exert the high level of effort[10] is solution of:

$$\max_{\Pi_1, \Pi_2^l, \Pi_2^h} \quad (R - \Pi_1) - \overline{\psi} + \overline{p}u\left(R - \Pi_2^l\right) + (1 - \overline{p})u\left(R - \Pi_2^h\right) \tag{3.5}$$
$$\text{s.t.} \quad \begin{cases} \Pi_1 + \overline{p}\Pi_2^l + (1 - \overline{p})\Pi_2^h \geq \overline{K} \\ u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \dfrac{\psi}{\Delta p} \end{cases}$$

The contract solution of this program then represents the overall optimum if it provides agents with more expected utility that the contract $(\underline{K} - \Pi_2^*, \Pi_2^*, \Pi_2^*)$, the optimal contract with low effort. We only focus in the following on the optimal incentive compatible contract.

[10] The generalization to continuous effort seems difficult as it would introduce marginal utility in the program through a two-step optimization. However, our model seems to be easy generalizable to a finite number of effort levels. As we focus on the incentive to exert the maximal level of effort, the contract would be incentive compatible if for each level of effort, the benefit of exerting the highest effort outweighs the cost. The binding incentive constraint would then correspond to the level of effort that has the highest cost-benefit ratio.

We don't discuss the issue of the optimal level of effort and rather assume that it is optimal for all agents to exert the maximal level of effort[11].

The experienced reader should note that the program (3.5) can be easy related to a principal-agent model with outside option effect that can be formulated (in a two-state, two-level of effort case) as:

$$\max_{X_1,X_2^l,X_2^h} \quad X_1 \tag{3.6}$$

$$\text{s.t.} \quad \begin{cases} X_1 + \overline{p}X_2^l + (1-\overline{p})X_2^h \leq \overline{W} \\ u(X_2^l) - u(X_2^h) \geq \dfrac{\psi}{\Delta p} \\ \overline{p}u(X_2^l) + (1-\overline{p})u(X_2^h) \geq u(Y) \end{cases}$$

where X_1 represent the principal payoff; X_2^l and X_2^h the revenues of the agent in two states that occur with respective probability $\overline{p}$ and $(1-\overline{p})$ when the agent exerts the effort at a cost of utility ψ; $\overline{W}$ the total wealth and Y the outside option of the agent. The result of Thiele and Wambach [67], induces in the setting that if $P > 3A$ - and consequently if the agent is farsighted - under unobservable efforts makes the incentive scheme less expensive for the principal $\left(\dfrac{\partial X_1}{\partial Y}\text{ is higher under moral hazard than if effort is observable}\right)$.

Foresight will also play a preponderant role in our setting when analyzing the impact of moral hazard. The solution of program (3.5) is defined by:

$$\begin{cases} u(R - \Pi_2^{l\,**}) - u(R - \Pi_2^{h\,**}) = \dfrac{\psi}{\Delta p} \\ \dfrac{\overline{p}}{u'\left(R - \Pi_2^{l\,**}\right)} + \dfrac{1-\overline{p}}{u'\left(R - \Pi_2^{h\,**}\right)} = 1 \\ \Pi_1^{**} = \overline{K} - \overline{p}\Pi_2^{l\,**} - (1-\overline{p})\Pi_2^{h\,**} = \overline{K} - E(\Pi_2^{**}) \end{cases} \tag{3.7}$$

[11] In a static model, Jullien et al. [40] gives condition under which more risk-averse agents optimally exert more effort of secondary and tertiary prevention (self-insurance and self-protection)

Therefore, if $(1/u')$ is concave (resp. convex), the optimal incentive contract under moral hazard satisfies $\dfrac{1}{u'\left(R-\overline{K}+\Pi_1^{**}\right)}=\dfrac{1}{u'\left(R-E\left(\Pi_2^{**}\right)\right)}>\dfrac{\overline{\pi}}{u'\left(R-\Pi_2^{l\,**}\right)}+\dfrac{1-\overline{\pi}}{u'\left(R-\Pi_2^{h\,**}\right)}=$ $1\left(\text{resp. } \dfrac{1}{u'\left(R-\overline{K}+\Pi_1^{**}\right)}<1\right)$.

As, under observable effort, $\dfrac{1}{u'(R-\overline{K}+\Pi_1^{*})}=\dfrac{1}{u'(R-\Pi_2^{*})}=1$, the next proposition hold.

Proposition 3.1 *If agents are farsighted (resp. not farsighted) in second period, moral hazard enhances (resp. reduces) prepayment of premia* (*as then* $\Pi_1^{**}>\Pi_1^{*}$, *resp.* $\Pi_1^{**}<\Pi_1^{*}$).

Proposition 3.1 can be linked with the finding of Gollier et al. [32]. They state that if $P\geq 2A$, better information structure reduces the efficient level of consumption in first period. They define a "better information structure" as a more dispersed probability distribution. Therefore, it corresponds to more uncertainty. In our setting, moral hazard also correspond to more uncertainty as under observable effort, agents have the same level of wealth in second period whatever their type. As in Gollier et al. [32] if $P\geq 2A$, this leads to a decrease in first period consumption through an increase in the premium paid in first stage. Gollier et al. [32] interpret this condition as coming from two conflicting effects that arises from an increase in uncertainty. First, the increase in uncertainty decreases first period consumption because of 'precautionary motives'. This effect is more important the larger the index of absolute produce P introduced by Kimball [42]. However, as agents are risk adverse, the increase in uncertainty reduces expected wealth in second period. Therefore, it increases the marginal value on first period revenue and thus tends to reduce prepayment. The intensity of this effect is reflected by the index of absolute risk aversion A. They state that the first effect dominates if the index of absolute prudence is larger than twice the index of absolute risk aversion, that is if agents are farsighted.

In our context, the condition $P \geq 2A$ can also be related to the moral hazard concerns. First, the uncertainty during second period leads to prepayment of premia (that may be linked with precautionary saving) if prudence is high. In our setting, through the zero profit condition, this increases average wealth in second period. Then, because of the concavity of the utility function, the optimal contract has to exhibit a higher classification risk (a higher spread between second period premia) to remain incentive compatible. This last effect goes against an increase in first period premium, and dominates if agents are "too risk adverse" relatively to their prudence. Proposition 3.1 states that the will be the case if $\frac{A}{P} \geq \frac{1}{2}$.

Now turn to second period premia. The first order condition gives $\frac{\overline{p}}{u'\left(R-\Pi_2^{l\,**}\right)} + \frac{1-\overline{p}}{u'\left(R-\Pi_2^{h\,**}\right)} = \frac{1}{u'\left(R-\Pi_2^{*}\right)}$. Moreover, the incentive constraint implies $\Pi_2^l < \Pi_2^h$. This leads to the following proposition.

Proposition 3.2 *Whatever the extent of prepayment of premia when young, the unobservability of effort improves the welfare of low risk agents and worsens the welfare of high risk agents when old. Therefore, it increases classification risk.*

This result is mainly driven by the incentive scheme. The first order condition leads to decreasing relationship between Π_2^l and Π_2^h and is satisfied at the first best contract $\left(\Pi_2^l = \Pi_2^h = \Pi_2^*\right)$. Now, to be incentive compatible, the optimal contract necessarily specifies $\Pi_2^l < \Pi_2^h$. Therefore, at the optimum $\Pi_2^l < \Pi_2^* < \Pi_2^h$.

It is worthwhile to note that second period premia are fully determined by the first order condition and the incentive constraint. The feasibility constraint then determines the premium paid when young depending on expected second period premia. This allows analyzing the solution graphically. To do so let us recall $X_1 \equiv R - \Pi_1 - \overline{\psi}$, $X_2^l \equiv R - \Pi_2^l$, $X_2^h \equiv R - \Pi_2^h$ and study the optimal premia in the plan (X_2^l, X_2^h).

Consider first the incentive constraint $u\left(X_2^l\right) - u\left(X_2^h\right) = \frac{\psi}{\Delta p}$. In the plan (X_2^l, X_2^h) it defines an increasing and concave curve below the 45-degree line (labeled IC in Figure 3.4). Moreover, the distance between the incentive constraint and the 45-degree line is increasing in X_2^l as the function $f\left(X_2^l\right) = X_2^l - u^{-1}\left(u\left(X_2^l\right) - \frac{\psi}{\Delta p}\right)$ is increasing in X_2^l when $u'\left(X_2^l\right) < u'\left[u^{-1}\left(u\left(X_2^l\right) - \frac{\psi}{\Delta p}\right)\right] = u'\left(X_2^h\right)$.

In the plan $\left(X_2^l, X_2^h\right)$ the first order condition $\dfrac{\overline{p}}{u'\left(R - \Pi_2^{l**}\right)} + \dfrac{1-\overline{p}}{u'\left(R - \Pi_2^{h**}\right)} = 1$ (labeled FOC in Figure 3.4), corresponds to a decreasing curve going through point (X_2^*, X_2^*). It is moreover tangent to the line $pX_2^l + (1-p)X_2^h = X_2^*$ at (X_2^*, X_2^*), convex is agents are farsighted and concave otherwise.

On the basis of those two curves, we can then infer the first period wealth with the zero-profit condition that can be written as $E(X_2) = -X_1 + 2R - (\overline{K} + \overline{\psi})$. This effect is represented in Figure 3.4 through the line $E(X_2) = c$.

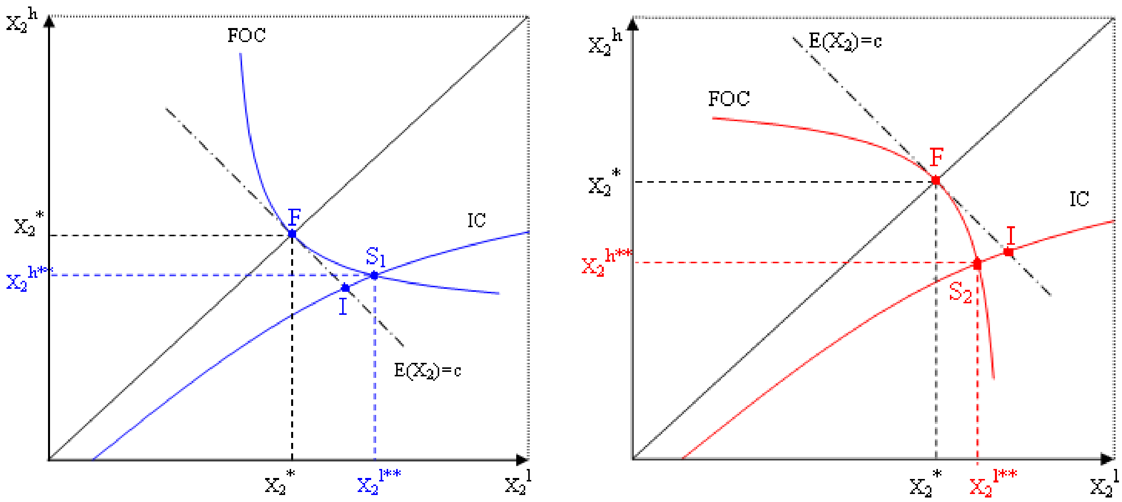


Figure 3.4: The impact of foresight on second period premia

This graphical analysis highlights that, depending on foresight, moral hazard has different effects on second period premia. It first appears that whatever the concavity of $1/u'$, an "incentive" effect leads - relative to the complete information benchmark - to an increase in wealth for low risk agents (decrease in Π_2^l) and a decrease in wealth for high risk agents. Graphically this corresponds to a move along the line $E(X_2) = c$ from point F to point I. This effect is combined with a "foresight" effect that depends on the concavity of $1/u'$. Indeed, if $1/u'$ is concave (left hand side figure) the "incentive" effect is coupled with a move to the north-east along the incentive constraint from point I to S_1. Therefore, when agents are farsighted, the "foresight" effect corresponds to a decrease in both second period premia. This last effect moreover leads to the increase in prepayment described in proposition 3.1, as it move the line $E(X_2) = c$ upward.

The reverse effect (represented by a move from I to S_2) holds when $1/u'$ is convex (right hand side figure). The "foresight" effect then corresponds to a decrease in both X_2^l and X_2^h (that leads to the increase in X_1 found in Proposition 3.1). However, as the first order condition is decreasing, the "incentive" effect always dominates and Proposition 3.2 holds.

Propositions 3.1 and 3.2 confirm our interpretation in term of foresight of the condition $P \geq 2A$. As shown in Crainich and Eeckhoudt [15], prudence, as well as risk aversion, can be interpreted as a preference toward risk. Here we show how these two kinds of preferences can be linked. In our setting, agents can use two mechanisms to reduce classification risk in second period. Either they can exert a preventive effort that reduces the probability of becoming high risk, or they can transfer wealth from period 1 to period 2 though prepayment of premia. Via an analogy to precautionary saving, this last mechanism can be linked with the notion of prudence, whereas the latter one is related to risk aversion.

Here we show that the preference for one mechanism to another is driven by the ratio absolute prudence to absolute risk aversion and more precisely, by the position of the ratio with respect to a critical level equal to 2.

As in the literature we discussed in section 3.2, the condition $P - 2A \geq 0$ therefore comes from a trade-off between a decrease in present consumption and an increase in preventive effort, that arises from second period uncertainty (here moral hazard generates classification risk). If agents are farsighted (that is if $P \geq 2A$), they prefer to transfer wealth from period 1 to period 2 rather than to exert effort, when they face classification risk. In this sense they rather foresee than prevent. In our setting this materialized therefore in an increase in front-loading ($\Pi_1^{**} > \Pi_1^*$) and a more difficult incentive scheme. It is then necessary to specify a large spread between second period premia to make sure that farsighted agents exert the effort (see Figure 3.4). The reverse effort holds for non farsighted agents that rather exert preventive effort than transfer wealth. As transferring wealth correspond in our setting to an increase in insurance against second period health cost, non farsighted agents' preferences correspond to the adage "an once of prevention is worth than a pound of cure". In this case, lower classification risk is therefore incentive compatible and uncertainty reduces prepayment of premia.

It seems important to note here that the reluctance of farsighted agents to exert effort, does not come from time inconsistency (as it can for example arise from beta-delta preferences a la Laibson [44]). Indeed, farsighted agents are perfectly time consistent but do not choose preventive effort (but rather wealth transfers) to face future uncertainty. This behavior may be caused by the uncertain nature of prevention relative to the predetermined (by the insurance contract) returns of prepayment.

3.5 Comparative Statics: How to Reduce Classification Risk?

In this section, we analyze the effect of the different parameters of the model on optimal premia. By defining which variables affect classification risk, we are then able to formulate some policy recommendations on the ways to reduce this kind of risk that produces inequalities. Our model contains three classes of variables: the variables regarding the income process (the sure revenue R and the expected health cost K_i^j), those regarding the preventive effort (the cost of effort ψ and the probabilities of being low type for both level of effort $\overline{p}$ and $\underline{p}$) and the variables relative to the behavior toward risk (the degrees of foresight, prudence and risk aversion) included in the utility function.

It is firstly worthwhile to note that – mainly because of the hypothesis of linear first period utility – the variables relating to the income process play a minor role in the determination of optimal premia. There is indeed no wealth effect in our model in the sense that all premia are proportional to sure revenue $\left(\frac{d\Pi_2^l}{dR} = \frac{d\Pi_2^h}{dR} = -\frac{d\Pi_1}{dR} = 1\right)$. Moreover, the expected costs of health only impact positively the first period premium (through the zero profit condition) and have no influence on reclassification risk.

3.5.1 Reduce the Cost of Primary Preventive Effort

The cost of primary preventive effort appears to be a first tool on which policymaker (using subsidizes) or insurer (as in the example presented in the introduction) may act. Through the system (3.7), our model allows to analyze of the impact of this cost ψ on optimal premia and especially on reclassification risk.

To do so let us differentiate the solution system with respect to Π_1, Π_2^l, Π_2^h and ψ. This leads to:

$$\begin{cases} -u'(R-\Pi_2^l)d\Pi_2^l + u'(R-\Pi_2^h)d\Pi_2^h = \frac{d\psi}{\Delta p} \\ \overline{p}\dfrac{u''\left(R-\Pi_2^l\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^2}d\Pi_2^l + (1-\overline{p})\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^2}d\Pi_2^h = 0 \\ d\Pi_1 + \overline{p}d\Pi_2^l - (1-\overline{p})d\Pi_2^h = 0 \end{cases}$$

With the first two equations one gets

$$\begin{cases} \dfrac{d\Pi_2^h}{d\psi} = \Delta p \dfrac{\overline{p}u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^2}{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^3 + \overline{p}u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^3} > 0 \\ \dfrac{d\Pi_2^l}{d\psi} = -\Delta p \dfrac{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^2}{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^3 + \overline{p}u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^3} < 0 \end{cases}$$

and therefore

$$\frac{d\Pi_1}{d\psi} = \overline{p}\,(1-\overline{p})\,\Delta p \frac{u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^2 - u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^2}{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^3 + pu''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^3}$$

This allows us to formulate the following proposition on the impact of a decrease in ψ

Proposition 3.3 *A decrease in the cost of tertiary prevention*

- *decreases classification risk (as it increases the premium paid by low risk and decreases the premium paid by high risk agents)*
- *decreases prepayment if agents are farsighted (increases it otherwise)*

The effect on classification risk is quite straightforward. A decrease in the cost of prevention enhances the incentive to exert the effort. Therefore, the insurance contract can exhibit a lower classification risk and remain incentive compatible. Therefore, if a policymaker wants to reduce the inequality resulting from classification risk he should work on reducing the cost of prevention. By the same mechanism as for Proposition 3.1, this will then decrease (resp. increase) prepayment if $P > 2A$ (resp. $P < 2A$). These effects can also be displayed graphically as an increase in ψ corresponds to a downward shit of the incentive curve.

Moreover, the same effects can be induced by a decrease in the probability of being low risk when not exerting the effort: $\underline{p}$. These changes in $\underline{p}$ are however difficult to interpret as it corresponds to a change in the probability for the agents that don't exert the effort, keeping the probability for those that exert the effort constant.

3.5.2 Increase the Effectiveness of Primary Prevention

The role of the probability of being low risk when exerting the preventive effort is more easily understandable. Indeed, an increase in $\overline{p}$, keeping $\underline{p}$ constant, can be interpreted as an improvement of the effectiveness of prevention. For example, investing in research on primary prevention, can increase the probability for the effort to lead to low risk type. It therefore opens a new spectrum for public policy.

Using (3.7), comparative statics on changes in $\overline{p}$ gives

$$\begin{cases} \dfrac{d\Pi_2^l}{d\overline{p}} = -\dfrac{\dfrac{1}{u'\left(R-\Pi_2^l\right)} - \dfrac{1}{u'\left(R-\Pi_2^h\right)} - (1-\overline{p})\dfrac{\psi}{(\Delta p)^2}\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^3}}{u'\left(R-\Pi_2^l\right)\left[\overline{p}\dfrac{u''\left(R-\Pi_2^l\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^3} + (1-\overline{p})\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^3}\right]} > 0 \\ \dfrac{d\Pi_2^l}{d\overline{p}} = -\dfrac{\dfrac{1}{u'\left(R-\Pi_2^l\right)} - \dfrac{1}{u'\left(R-\Pi_2^h\right)} + \overline{p}\dfrac{\psi}{(\Delta p)^2}\dfrac{u''\left(R-\Pi_2^l\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^3}}{u'\left(R-\Pi_2^l\right)\left[\overline{p}\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^3} + (1-\overline{p})\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^3}\right]} \end{cases}$$

This leads to

Proposition 3.4 *An increase in the probability of being low risk type in second period when exerting the preventive effort*

- *increases the optimal premium paid by low risk agents in second stage*
- *decreases classification risk if agents are farsighted*

First, an increase in the probability of being low risk for agents that exert the preventive effort (keeping constant this probability for agents that don't exert effort) increases the benefit of effort. It is then easier to provide the incentive to exert effort. The optimal contract can therefore lead to a lower welfare in good state of nature and still be incentive compatible.

Through the incentive constraint, this implies a decrease in second period premium for risky agents. However, the increases in $\overline{p}$ also decreases the weight attached to this bad state in the objective function. This leads to a decrease in optimal wealth of high risk. The combination of these two effects is ambiguous. Therefore, the effect of an increase in $\overline{p}$ on the expected wealth in second period is also ambiguous and we can not conclude on the impact of this increase on first period wealth.

However, it is possible to state that an increase in the probability of being low risk decreases the risk of classification when agents are farsighted:

$$\frac{d\left(X_2^l-X_2^h\right)}{d\overline{p}}=\frac{\left[\frac{1}{u'\left(X_2^l\right)}-\frac{1}{u'\left(X_2^h\right)}\right]^2+\frac{\overline{p}}{u'\left(X_2^l\right)u'\left(X_2^h\right)}\frac{\psi}{(\Delta p)^2}\left[\frac{u''\left(X_2^h\right)}{\left[u'\left(X_2^h\right)\right]^3}-\frac{u''\left(X_2^l\right)}{\left[u'\left(X_2^l\right)\right]^3}\right]-\frac{\psi}{\Delta p}\frac{u''\left(X_2^h\right)}{\left[u'\left(X_2^h\right)\right]^3u'\left(X_2^l\right)}}{\overline{p}\frac{u''\left(X_2^l\right)}{\left[u'\left(X_2^l\right)\right]^3}+(1-\overline{p})\frac{u''\left(X_2^h\right)}{\left[u'\left(X_2^h\right)\right]^3}}$$

Therefore, if agents are farsighted, the policymaker can reduce the classification risk by improving the effectiveness of primary prevention, for example through investments in medical research.

3.5.3 Comparative foresight

In the previous sections we have highlighted the important role of foresight on the determination of optimal premia. Whether agents are farsighted or not, widely impacts the consequences moral hazard has on premia. This raises the question of the influence of changes in the degree of foresight, that is in the concavity of $1/u'$. To study this issue let us first define formally the degree of foresight.

Definition 3.3 *An agent v with second period utility function $v(.)$, will be said to have a higher degree of foresight (or to be more farsighted) than agent u with second period utility function $u(.)$ if $1/v'(.)$ is a concave transformation of $1/u'$.*

Remark 3.1 *The equivalent to state that v is more farsighted than u if $P_v(x) - 2A_v(x) \geq P_u(x) - 2A_u(x) \forall x$*

Proof:

$$\frac{1}{v'}\left(\left(\frac{1}{u'}\right)^{-1}(t)\right) = g(t) \text{ with } g \text{ increasing and concave}$$

$$\Rightarrow \quad g'(t) = \frac{v''\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)}{\left[v'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)\right]^2} \cdot \frac{\left[u'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)\right]^2}{u''\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)} = \frac{A_v\left(\left(\frac{1}{u_1'}\right)^{-1}(t)\right)u'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)}{A_u\left(\left(\frac{1}{u_1'}\right)^{-1}(t)\right)v'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)}$$

$$\Rightarrow \quad \text{g is concave if and only if } \frac{d}{dx}\log\left(\frac{A_v(x)u'(x)}{A_u(x)v'(x)}\right) \leq 0$$

$$\Leftrightarrow \quad \frac{A_v'(x)}{A_v(x)} + \frac{u''(x)}{u'(x)} - \frac{A_u'(x)}{A_u(x)} - \frac{v''(x)}{v'(x)} \leq 0$$

$$\Leftrightarrow \quad P_v(x) - 2A_v(x) \geq P_u(x) - 2A_u(x)$$

Comparative statics on foresight then comes to compare utility functions. The effect of an increase in foresight is therefore hard to grasp as it also implies changes in incentive constraint and in first best contract. Therefore, to explicit results on the effect of foresight on optimal dynamic contract, it is necessary to impose further assumptions on preferences.

3.6 Explicit Examples

3.6.1 The case of Constant Relative Risk Aversion

The first convenient way to specify the utility function is to assume that agents' preferences are represented by CRRA (Constant Relative Risk Aversion) utility functions. CRRA utility functions indeed exhibit some interesting properties. If

$$u(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{for } \gamma > 0, \gamma \neq 1 \\ ln(x) & \text{for } \gamma = 1 \end{cases}$$

$A(x) = \frac{\gamma}{x}$ and $P(x) = \frac{\gamma+1}{x}$. Therefore, if agents v and u are both characterized by CRRA utility function with respective parameters γ_v and γ_u, from remark 3.1, v is more farsighted than u if $\gamma_v < \gamma_u$. CRRA utility functions also exhibit the convenient feature of leading to the same first best contract $\left(\overline{K} - R + 1, R - 1, R - 1\right)$ whatever the parameter of risk aversion $(u'(X_2^*) = 1 \Leftrightarrow X_2^* = 1 \forall \gamma > 0)$.

Consider two types of agents characterized by two CRRA utility functions $v(.)$ and $u(.)$ such that $1/v' = \phi \circ (1/u')$ with $\phi(.)$ increasing and concave; that is agents of type v are more farsighted than agents of type u.

If policyholders are of type v the optimal contract $\left(\Pi_{1v}^{**}, {\Pi_{2v}^{l}}^{**}, {\Pi_{2v}^{h}}^{**}\right)$ has to satisfy

$$\begin{cases} \dfrac{\overline{p}}{v'\left(R - {\Pi_{2v}^{l}}^{**}\right)} + \dfrac{1-\overline{p}}{v'\left(R - {\Pi_{2v}^{h}}^{**}\right)} = 1 \\ v(R - {\Pi_{2v}^{l}}^{**}) - v(R - {\Pi_{2v}^{h}}^{**}) = \dfrac{\psi}{\Delta p} \\ \Pi_{1v}^{**} = \overline{K} - \overline{p}{\Pi_{2v}^{l}}^{**} - (1-\overline{p}){\Pi_{2v}^{h}}^{**} \end{cases}$$

whereas if agents are characterized by the utility function $u(.)$ the optimal incentive compatible contract is the triplet $\left(\Pi_{1u}^{**}, \Pi_{2u}^{l}{}^{**}, \Pi_{2u}^{h}{}^{**}\right)$ such that

$$\begin{cases} \dfrac{\overline{p}}{u'\left(R-\Pi_{2u}^{l}{}^{**}\right)}+\dfrac{1-\overline{p}}{u'\left(R-\Pi_{2u}^{h}{}^{**}\right)}=1 \\ u(R-\Pi_{2u}^{l}{}^{**})-u(R-\Pi_{2u}^{h}{}^{**})=\dfrac{\psi}{\Delta p} \\ \Pi_{1u}^{**}=\overline{K}-\overline{p}\Pi_{2u}^{l}{}^{**}-(1-\overline{p})\Pi_{2u}^{h}{}^{**} \end{cases}$$

Consider the relative position of the curve characterized by the first two equations of each system in the plan $\left(X_2^l, X_2^h\right)$.

Regarding the first order conditions, one of the properties of CRRA utility functions turns out to be crucial. All CRRA utility functions leading to the same first best contract we necessarily have $\frac{1}{v'(1)}=\phi\left(\frac{1}{u'(1)}\right)=\frac{1}{u'(1)}=1$ that is $\phi(1)=1$. Therefore

$$\begin{aligned} & \frac{\overline{p}}{v'\left(R-\Pi_{2v}^{l}{}^{**}\right)}+\frac{1-\overline{p}}{v'\left(R-\Pi_{2v}^{h}{}^{**}\right)}=\overline{p}\phi\left(\frac{1}{v'\left(R-\Pi_{2v}^{l}{}^{**}\right)}\right)+(1-\overline{p})\,\phi\left(\frac{1}{v'\left(R-\Pi_{2v}^{h}{}^{**}\right)}\right)=1 \\ \Rightarrow\ & \frac{\overline{p}}{v'\left(R-\Pi_{2v}^{l}{}^{**}\right)}+\frac{1-\overline{p}}{v'\left(R-\Pi_{2v}^{h}{}^{**}\right)}>\phi^{-1}(1)=1=\frac{\overline{p}}{u'\left(R-\Pi_{2u}^{l}{}^{**}\right)}+\frac{1-\overline{p}}{u'\left(R-\Pi_{2u}^{h}{}^{**}\right)} \end{aligned}$$

as ϕ is increasing and concave. The function $1/u'$ being increasing, this implies that in the plan $\left(X_2^l, X_2^h\right)$, the first order condition of agents v (labeled FOC_v in Figure 3.5) is higher than the one of agents u (labeled FOC_u), although they both have the same tangency at (X_2^*, X_2^*).

As stated in previous section this effect is however coupled with an effect on the incentive constraint. In the case of CRRA utility function, the incentive constraint becomes $\frac{(X_2^l)^{1-\gamma}}{1-\gamma}-\frac{(X_2^h)^{1-\gamma}}{1-\gamma}=\frac{\psi}{\Delta p}$. In the plan $\left(X_2^l, X_2^h\right)$, the slope of the incentive constraint writes $\frac{dX_2^h}{dX_2^l}=\left(\frac{X_2^h}{X_2^l}\right)^{\gamma}<1$ and is decreasing in γ. Therefore, an increase in foresight makes the slope of the incentive constraint turn counter-clockwise (from IC_u to IC_v in Figure 3.5). We are however unable to define when the incentive constraints of both individuals cross and the graphical analysis is not sufficient to conclude in this case.

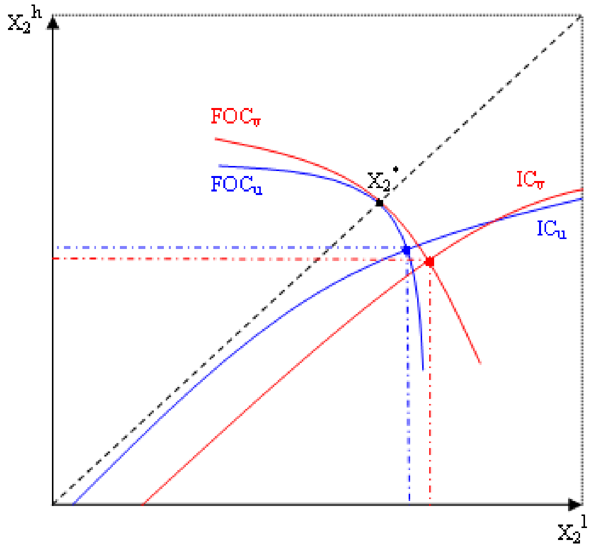


Figure 3.5: Comparative foresight in the case of CRRA preferences

Let us therefore turn to an analytical analysis.

With CRRA preferences, the optimal second period contract is described by the system

$$\begin{cases} \dfrac{\left(X_2^l\right)^{1-\gamma}}{1-\gamma} - \dfrac{\left(X_2^h\right)^{1-\gamma}}{1-\gamma} = \dfrac{\psi}{\Delta p} \\ \overline{p}\left(X_2^l\right)^{\gamma} + (1-\overline{p})\left(X_2^h\right)^{\gamma} = 1 \end{cases} \tag{3.8}$$

Therefore,

$$\begin{cases} \dfrac{dX_2^l}{d\gamma} = -\dfrac{\dfrac{\psi}{\Delta p(1-\gamma)} - \dfrac{1}{1-\gamma}\ln\left(X_2^l\right)\left(X_2^l\right)^{1-\gamma} + \dfrac{1}{\gamma(1-\gamma)}\ln\left(X_2^h\right)\left(X_2^h\right)^{1-\gamma} + \dfrac{\overline{p}}{1-\overline{p}}\dfrac{1}{\gamma}\ln\left(X_2^l\right)\left(X_2^l\right)}{\dfrac{1}{(1-\overline{p})}\left(X_2^l\right)^{\gamma-1}\left[(1-\overline{p})\left(X_2^l\right)^{1-2\gamma} + \overline{p}\left(X_2^h\right)^{1-2\gamma}\right]} \\ \dfrac{dX_2^h}{d\gamma} = \dfrac{\dfrac{\psi}{\Delta p(1-\gamma)} - \dfrac{1}{\gamma(1-\gamma)}\ln\left(X_2^l\right)\left(X_2^l\right)^{1-\gamma} + \dfrac{1}{1-\gamma}\ln\left(X_2^h\right)\left(X_2^h\right)^{1-\gamma} - \dfrac{1-\overline{p}}{\overline{p}}\dfrac{1}{\gamma}\ln\left(X_2^h\right)\left(X_2^h\right)^{\gamma}}{\dfrac{1}{\overline{p}}\left(X_2^h\right)^{\gamma-1}\left[(1-\overline{p})\left(X_2^l\right)^{1-2\gamma} + \overline{p}\left(X_2^h\right)^{1-2\gamma}\right]} \end{cases}$$

As $X_2^* = 1 \forall \gamma > 0$, from Proposition 3.2, $\ln\left(X_2^l\right) > 0$ and $\ln\left(X_2^h\right) < 0$. Therefore, if $\gamma > 1$, the three last terms of both numerators are positive and next proposition holds

Proposition 3.5 *If agents' preferences in second period are represented by a Constant Relative Risk Aversion utility function with risk coefficient larger than one, an increase in the degree of foresight*

- *decreases the premium paid by low risk agents*
- *increases the premium paid by high risk agents*
- *and thus increases classification risk*

in second period, provided the cost of effort is low enough (relative to its benefits)

Indeed, when $(1-\gamma) < 0$, provided the first term of the numerators is low enough relative to the other terms, the optimal wealth in second stage decreases with the coefficient of risk aversion γ when the agent turns out to low risk $\left(\frac{dX_2^l}{d\gamma} > 0\right)$ and increases with γ for high risk agents $\left(\frac{dX_2^h}{d\gamma} < 0\right)$. In the case of CRRA utility functions, the degree of foresight increases when γ decreases and Proposition 3.5 follows.

This result can be explained by the two effects induced by the increase in foresight already highlighted. First, the pure foresight effect, represented by the move of the first order condition in the plan (X_2^l, X_2^h) presented above, leads to a decrease in both second period premia (and an increase in first period premium). Proposition 3.5 states that the incentive effect highly depends on the cost of effort. This mainly comes from the influence of moves in γ on the level of utility reached for low level of wealth. Indeed, an increase in the coefficient of relative risk aversion has a large impact on low levels of utility, whereas the change in utility for high level of consumption is relatively small (cf. Gollier [30] Figure 2.2). This impacts the intensity of the counter-clockwise move of the incentive constraint. When the needed spread between wealth is low (low level of $\frac{\psi}{\Delta p}$), the incentive constraints of agents having different CRRA utility functions cross for high level of wealth. Therefore, the first order condition of more farsighted agents crosses their incentive constraint when it is below the incentive constraint of less farsighted agents (as in the case represented in Figure 3.5).

Then, the combination of the two effects leads to an increase in wealth of low risk agents and a decrease in wealth of high risk. However, when $\frac{\psi}{\Delta p}$ is too high, the incentive constraints of two agents may cross before crossing the first order condition, leading to the reverse effects (a decrease in X_2^l and an increase in X_2^h).

The restriction to non-farsighted agents ($\gamma > 1$) may seem awkward, but is pretty standard in the case of CRRA utility function. For example, Gollier [30] argues that this condition holds for most households in real economy. This moreover corresponds to the necessary condition for utility to be unbounded below, which is a standard assumption in principal-agents models (see for example Grossman and Hart [34]). It notably ensures that no nonnegativity constraints on income bind at the optimum.

Proposition 3.5 moreover have implications on first period premium. From the zero profit condition, $\frac{d\Pi_1}{d\gamma} = -\overline{p}\frac{d\Pi_2^l}{d\gamma} - (1-\overline{p})\frac{d\Pi_2^l}{d\gamma}$. Therefore, for high value of $\overline{p}$, the effect of low risk premium dominates. Then, in the configuration of proposition 3.5, the first stage premium increases in foresight. This is confirmed and generalized by the study of the system

$$\begin{cases} \dfrac{\left(X_2^l\right)^{1-\gamma}}{1-\gamma} - \dfrac{\left(X_2^h\right)^{1-\gamma}}{1-\gamma} = \dfrac{\psi}{\Delta p} \\ \overline{p}\left(X_2^l\right)^{\gamma} + (1-\overline{p})\left(X_2^h\right)^{\gamma} = 1 \\ X_1 + \overline{p}X_2^l + (1-\overline{p})X_2^h = 2R - \overline{K} \end{cases}$$

that gives

$$\begin{aligned} \frac{dX_1}{d\gamma} \quad = \quad & \Bigg\{ (1-\overline{p})^2 \left(X_2^h\right)^{2\gamma} ln\left(X_2^h\right) + \overline{p}^2 \left(X_2^l\right)^{2\gamma} ln\left(X_2^l\right) + \overline{p}(1-\overline{p})\frac{\gamma}{(1-\gamma)^2}\Big[\left(X_2^h\right)^{\gamma-\frac{1}{2}}\left(X_2^l\right)^{\frac{1}{2}} \\ & - \left(X_2^l\right)^{\gamma-\frac{1}{2}}\left(X_2^h\right)^{\frac{1}{2}}\Big]^2 + \frac{\overline{p}(1-\overline{p})}{1-\gamma}\Big(\ln\left(X_2^h\right) X_2^h \left(X_2^l\right)^{\gamma}\left[\left(X_2^h\right)^{\gamma-1} - \left(X_2^l\right)^{\gamma-1}\right] \\ & + \ln\left(X_2^l\right) X_2^l \left(X_2^h\right)^{\gamma}\left[\left(X_2^l\right)^{\gamma-1} - \left(X_2^h\right)^{\gamma-1}\right]\Big)\Bigg\} \left[\gamma\left(\overline{p}\left(X_2^l\right)^{2\gamma-1} + (1-\overline{p})\left(X_2^h\right)^{2\gamma-1}\right)\right]^{-1} \end{aligned}$$

The first term being the only negative one, it turns out that, when the probability of being high risk type $(1-\overline{p})$ is low enough, an increase in foresight increases the first period premium. Simulations with CRRA preferences (cf. Appendix) moreover points out that this seems to be the case whatever the probability of being high risk. The intuitive effects driving Proposition 3.1 therefore seems to be generalizable to changes in the degree of foresight.

We are unable to determine analytically the limit cost of effort in Proposition 3.5. However, this can be done when analyzing the welfare in the good state of nature. To do so let us consider the following change of variables. If we define $Y_2^l \equiv u(X_2^l)$ and $f \equiv u^{-1}$, we have $u(X_2^h) = Y_2^l - \frac{\psi}{\Delta p}$ and the system defining the second period premia simplifies in

$$H(Y_2^l) \equiv \overline{p} f'(Y_2^l) + (1-\overline{p})\, f'(Y_2^l - K) = 1 \tag{3.9}$$

In the case of CRRA preferences,

$$f\,(Y) = \exp\left[\frac{1}{1-\gamma}\ln\left((1-\gamma)Y\right)\right] \text{ and } f'\,(Y) = \exp\left[\frac{\gamma}{1-\gamma}\ln\left((1-\gamma)Y\right)\right]$$

(f') being increasing, it follows from (3.9) that

$$\text{sgn}\left(\frac{\partial Y_2^l}{\partial \gamma}\right) = -\text{sgn}\left(\overline{p}\frac{\partial f'}{\partial \gamma}(Y_2^l) + (1-\overline{p})\,\frac{\partial f'}{\partial \gamma}(Y_2^l - K)\right)$$

Then, as $\text{sgn}\left(\frac{\partial f'}{\partial \gamma}(Y)\right) = \text{sgn}\left(\ln\left((1-\gamma)\,Y\right) - \gamma\right)$, one gets

$$\frac{\partial f'}{\partial \gamma}(Y) \geq 0 \Leftrightarrow Y \leq Y_\gamma \equiv \frac{e^\gamma}{1-\gamma}\text{when}\gamma > 1$$

Therefore, a sufficient condition for the welfare in low state to be increasing with foresight $\left(\frac{\partial Y_2^l}{\partial \gamma} \leq 0\right)$, is Y_2^l (and thus $Y_2^l - K$) to be lower than Y_γ. As $H(Y_2^l)$ is increasing, this condition is equivalent to $H(Y_\gamma) \geq 1$, for which a sufficient condition is $f'(Y_\gamma - K) \geq 1$ that is $\dfrac{\psi}{\Delta p} \leq \dfrac{e^\gamma - 1}{1-\gamma}$.

Proposition 3.6 *If agents' preferences in second period are represented by a Constant Relative Risk Aversion utility function with risk coefficient larger than one, an increase in foresight increases the welfare of low risk agents in second period, if* $\dfrac{\psi}{\Delta p} \leq \dfrac{e^\gamma - 1}{1-\gamma}$.

We however can not infer results on level of wealth (and thus on premia) from this proposition as the coefficient of relative risk aversion doesn't have a monotonic impact on utility derived from a given level of wealth (see Gollier [30], Figure 2.2).

3.6.2 A suitable utility function with linear reciprocal derivative

To go further in the analysis of the impact of foresight on premia and have clearer results, it is accommodating to build a suitable utility function that satisfies additional simplifying properties.

From equation (3.9), it first seems simplifying to specify linear $f'(y)$, that is $f'(y) = \theta y$ where θ depicts the behavior toward risk. As, $f = u^{-1}$, this corresponds to $u(x) = \frac{1}{\theta}\sqrt{2\theta(x-a)}$ where a represents the constant of integration. Then, to isolate the effect foresight, it appears useful to consider a class of utility function for which (as in the CRRA case) the first best premium doesn't depend on the behavior toward risk parameter. To do so let us specify the constant a such that $u'(1) = 1 \forall \theta$ that is $a = 1 - \frac{1}{2\theta}$. Let us therefore consider the class of utility function:

$$u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1} \quad , \quad \theta > 0 \tag{3.10}$$

which is twice continuously differentiable, increasing and concave above some (subsistence) level [12] $\underline{x} \equiv 1 - \frac{1}{2\theta}$ and satisfies the Inada conditions $\lim_{x \to \underline{x}} u'(x) = +\infty$ and $\lim_{x \to +\infty} u'(x) = 0$.

An agent whose preferences is described by (3.10) is risk adverse $\left(A = \dfrac{\theta}{2\theta(x-1)+1} > 0 \right)$, prudent $\left(P = \dfrac{3\theta}{2\theta(x-1)+1} > 0 \right)$ and farsighted $\left(P - 2A = \dfrac{\theta}{2\theta(x-1)+1} > 0 \right)$ for all level of θ (positive). Moreover, if we consider two agents u and v having such preferences, v is more farsighted than u ($P_v - 2A_v > P_u - 2A_u \ \forall x > \underline{x}$) if and only if $\theta_v > \theta_u$.

The levels of wealth reached under the optimal incentive contract $\left(\text{that exists only if } \dfrac{\psi}{\Delta p} < \dfrac{1}{\overline{p}\theta} \right)$ then writes:

$$\left\{ \begin{array}{lcl} X_2^{l**} & = & 1 + (1-\overline{p}) \dfrac{\psi}{\Delta p} + \dfrac{1}{2} \left((1-\overline{p}) \dfrac{\psi}{\Delta p} \right)^2 \theta \\ X_2^{h**} & = & 1 - \overline{p} \dfrac{\psi}{\Delta p} + \dfrac{1}{2} \left(\overline{p} \dfrac{\psi}{\Delta p} \right)^2 \theta \\ X_1^{**} - X_1^{*} & = & 1 - E\left(X_2^{**}\right) = -\dfrac{1}{2} \overline{p} \left(1-\overline{p}\right) \left(\dfrac{\psi}{\Delta p} \right)^2 \theta \end{array} \right.$$

Therefore,

Proposition 3.7 *If agents' preferences are defined by* $u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1}$, $\theta > 0$, *an increase in the degree of foresight*

(i) increases first period premium

(ii) decreases second period premia

(iii) increase classification risk if the good state of health is the more likely ($p > 1/2$)

The use of a utility function whose reciprocal has a linear derivative allows us to better isolate the impact of foresight. We can indeed relate the results of Proposition 3.7 with the effect highlighted in Proposition 3.1. An increase in foresight (that is in $P - 2A$) increases the first period premium for precaution motive (increase in P) and/or because agents are less sensitive to the increase in classification risk it may cause (decrease in A).

[12] In this sense, this class of utility functions can be related to the Stone-Geary class

The incentive effect of such an increase does not prevent - for this class of utility function - from a decrease in both second period premia.

Moreover, the increase in wealth is higher in the most probable state and therefore this increase in the first period may not be coupled with an increase in classification risk if the bad state is highly probable. The optimal incentive contract moreover confirms the result of Proposition 3.3 and 3.4 as here (i) an increase in the cost of effort increases the first period premium and the classification risk (when the optimal contract exists, that is when $\frac{\psi}{\Delta p} < \frac{1}{\overline{p}\theta}$) and (ii) an increase in the probability of being low risk when exerting the effort decreases the classification risk and the optimal wealth in the good state.

3.7 The introduction of short-term (spot) insurance

Hendel and Lizzeri [36] have pointed out that, in the absence of moral hazard, dynamic insurance contracts are subject to lapsation in second period. Taking the feature into account, the timing our model becomes

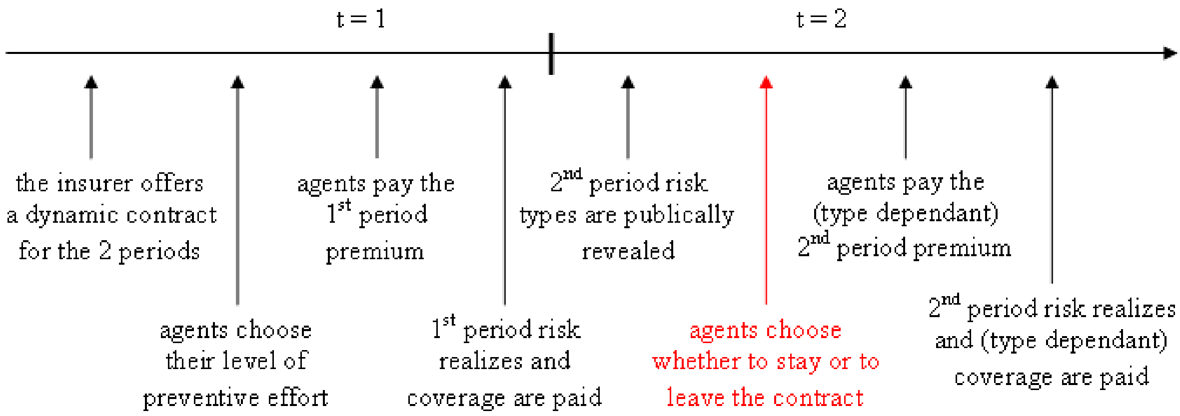


Figure 3.6: The timing of the game including interim participation choice

In the absence of severance payment (proposed by Cochrane [14]), healthier agents may then leave the dynamic contract to go to a competing short-term (spot) insurer that offers actuarially fair premia. This will be the case if the optimal incentive premium for low risk agent is higher than their expected health cost $\left(\Pi_2^{l\,**} > K_2^l\right)$. The second period optimal contract presented previously however does not depend on expected costs (the expected health costs over the two periods only influences the first period premium). We can still infer that for a given expected health costs schedule, the lower the premium offered by the mutual dynamic insurer, the lower the incentive for healthy agents to lapse (and go to the spot insurer).

Therefore, from Proposition 3.2, the mutual insurer described here suffers from less lapsation if effort is unobservable. Moreover, from Proposition 3.5 and 3.7:

Proposition 3.8 *A mutual insurer that offers long-term contracts is more likely to be sustainable to the competition of companies offering spot (short-term) contracts if it insures more farsighted agents, when agents preferences*

- *are described by* $u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1}$, $\theta > 0$
- *are CRRA, provided the cost of primary preventive effort is low enough*

Our work also offers an alternative explanation to the various degrees of front-loading and of lapsation observed in dynamic insurance contracts. In the empirical part of their work, Hendel and Lizzeri [36] show that in life insurance, more front-loading is associated with lower lapsation. They then argue that this phenomenon can be explained by heterogeneity in agents' income growth (that is in the cost of front-loading). It appears from our model that this can also be explained by heterogeneity in foresight. Program (3.5) indeed also fits with the problem of a competitive insurance company that does not discount the future (in the zero profit condition (3.2)). In this case, it appears from our study that the co-existence of dynamic contract with spot contract, and the various degree of front-loading may be explained by moral hazard and heterogeneous foresight.

Moreover, contracts of more farsighted agents may exhibits more front loading and less lapsation if the reciprocal of agents' utility function has a linear derivative ($u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1}$, $\theta > 0$). Simulations with CRRA utility function (cf. Appendix) seems to indicate that this would also be the case when agents have CRRA preferences if the cost of primary preventive effort is low enough.

3.8 Allowing for Cross Subsidization: A Step Toward Adverse Selection

A natural extension of our model consists in allowing for cross subsidization between agents with heterogeneous preferences. Let us therefore study a situation where a mutual insurer dynamically insures a population composed of two type of agents with respective utility functions u and v and respective proportion λ and $1-\lambda$. Assuming the insurer is unable to distinguish between types and can only offer a single (pooling) contract, the optimal incentive compatible contract is solution of

$$\begin{aligned}
\max_{\Pi_1,\Pi_2^l,\Pi_2^h} \quad & (R-\Pi_1) - \overline{\psi} + \lambda\left[\overline{p}u\left(R-\Pi_2^l\right) + (1-\overline{p})u\left(R-\Pi_2^h\right)\right] \\
& + (1-\lambda)\left[\overline{p}v\left(R-\Pi_2^l\right) + (1-\overline{p})v\left(R-\Pi_2^h\right)\right] \\
\text{s.t.} \quad & \begin{cases} \Pi_1 + \overline{p}\Pi_2^l + (1-\overline{p})\Pi_2^h \geq \overline{K} \\ u(R-\Pi_2^l) - u(R-\Pi_2^h) \geq \dfrac{\psi}{\Delta\pi} \\ v(R-\Pi_2^l) - v(R-\Pi_2^h) \geq \dfrac{\psi}{\Delta\pi} \end{cases}
\end{aligned}$$

To guarantee that only one incentive constraint will bind at the optimum, let us moreover assume that agents of type v have a strictly lower preference for future, that is $v'(x) < u'(x)$ $\forall x$. This guarantees that the incentive constraint of agents with preferences u doesn't bind at the optimum ($u(R-\Pi_2^l) - u(R-\Pi_2^h) > v(R-\Pi_2^l) - v(R-\Pi_2^h)$ $\forall(\Pi_2^l, \Pi_2^h)$).

Then, the first order conditions of the program can be written as:

$$\frac{\overline{p}}{v'\left(R-\Pi_2^l\right)}+\frac{1-\overline{p}}{v'\left(R-\Pi_2^h\right)}=\lambda+(1-\lambda)\left(\overline{p}\frac{u'\left(R-\Pi_2^l\right)}{v'\left(R-\Pi_2^l\right)}+(1-\overline{p})\frac{u'\left(R-\Pi_2^h\right)}{v'\left(R-\Pi_2^h\right)}\right)$$

Reminding that we assume $v'(x) < u'(x)$, this gives $\frac{\overline{p}}{v'\left(R-\Pi_2^l\right)}+\frac{1-\overline{p}}{v'\left(R-\Pi_2^h\right)} > 1$. As the incentive constraint of agents of type v always binds at the equilibrium, the following proposition hold

Proposition 3.9 *Allowing for cross subsidization between heterogeneous agents*

- *increases prepayment*
- *decreases second period premia*
- *increases classification risk*

of agents with the lowest preference for future

By definition, the pooling contract provides arbitration between the preferences of the two types of agents. In our setting, the agents with the lower preference for future are the most difficult to incite and therefore drive the incentive compatibility of the contract. Reminding that in the plan (X_2^l, X_2^h) the incentive compatible constraint defines an increase curve that goes away from the 45° line as X_2^l increase, Proposition 3.9 holds. Then, cross-subsidization has two main implications for the agents with the lower preference for future. First, as it increases their second period wealth (in both states), cross-subsidization lowers their incentive to leave the contract when they turn out to be low risk. It therefore stabilizes the dynamic contract. This is however done at the cost of an increase in classification risk, for this type of agents at least. Without further assumptions, we are unable to infer the impact of cross-subsidization on the welfare of agents of type u (with a higher preference for future), as it would highly depend on the relative degree of foresight of both type. The introduction of adverse selection in our setting remains therefore an open issue that may call for some refinement in the model, as the non-linearity of first period preferences or the full specification of the second period utility function.

3.9 Applications to other insurance markets

The model presented above in the case of health insurance seems to be applicable to other insurance markets with slight modifications.

3.9.1 Life insurance

A first application concerns life insurance. The considered insurer then offers a protection against the risk of death, and agents can reduce the probability of having a high probability of death in second period by exerting preventive effort. If we assume ad-hoc altruism (in the sense that the indemnity paid to the beneficiary directly enters in the insured's utility function), optimal insurance is complete in each state and is solution of a program similar to 3.5. We however need to introduce in this extension the fact that agents can die in first period (that is the survival probability). As q_1 represents here the risk of death in period 1, only a portion $(1 - q_1)$ of a generation is still alive in period 2. This effect enters the objective, the incentive constraint and the zero profit condition such that the program becomes

$$\max_{\Pi_1,\Pi_2^l,\Pi_2^h} \quad (R - \Pi_1) - \overline{\psi} + (1 - q_1)\left[\overline{p}u\left(R - \Pi_2^l\right) + (1 - \overline{p})u\left(R - \Pi_2^h\right)\right] \tag{3.11}$$

$$\text{s.t.} \quad \begin{cases} \Pi_1 + (1 - q_1)\left[\overline{p}\Pi_2^l + (1 - \overline{p})\Pi_2^h\right] \geq \overline{K} = q_1 R + (1 - q_1)\left[\overline{p}q_2^l R + (1 - \overline{p})q_2^h R\right] \\ u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \dfrac{\psi}{(1 - q_1)\Delta p} \end{cases}$$

The solution of this new program, very similar to original one, is given by:

$$\begin{cases} u(R - \Pi_2^{l**}) - u(R - \Pi_2^{h**}) = \dfrac{\psi}{(1 - q_1)\Delta p} \\ \dfrac{\overline{p}}{u'\left(R - \Pi_2^{l**}\right)} + \dfrac{1 - \overline{p}}{u'\left(R - \Pi_2^{h**}\right)} = \dfrac{1}{1 - q_1} \\ \Pi_1^{**} = \overline{K} - \overline{p}\Pi_2^{l**} - (1 - \overline{p})\Pi_2^{h**} = \overline{K} - E(\Pi_2^{**}) \end{cases} \tag{3.12}$$

Then, all the above properties hold in the case of life insurance with (ad-hoc) altruistic agents. Particularly, long term life insurance appears more stable relative to spot insurance as it insures more farsighted agents. Moreover, the effect of q_1 turns out to be ambiguous. If the probability of dying during the first period decreases the relative valuation of the second period (and therefore tends to decrease inter-generational insurance), it also reduces the proportion of agents that shares the prepaid premia in second period (and then leads to an increase in second period wealth).

3.9.2 Unemployment insurance

With more amendments, our model also seems to be applicable to unemployment insurance.

Consider a social unemployment insurance in our simple overlapping generation model. In their early part of life, all agents face the same probability of being unemployed (or have the same expected length of unemployment) and can invest in training effort e. Partially based on this effort, agents can then either be employed as a "skilled" (executive) or "unskilled" (non executive) worker in second period. We moreover assume (as it seems to be the case in real economies) that the risk of unemployment is higher among unskilled workers than among skilled ones. Modeling the fact that the three types of agents also differ in wages, the income profile without insurance can be summarized as follows:

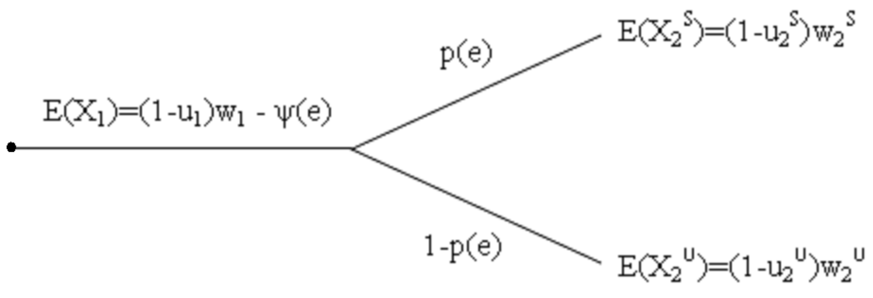


Figure 3.7: The income profile without unemployment insurance

with u_1, u_2^S and u_2^U the respective probabilities of being unemployed for youth, skilled and unskilled workers ; and w_1, w_2^S and w_2^U the respective wages of youth, skilled and unskilled workers.

As in our baseline model, it is then optimal for the mutual (social) insurer to provide risk-adverse agents with a complete insurance in each state that is with a triplet of sure consumption profiles C_1, C_2^S et C_2^U solution of

$$\max_{C_1, C_2^S, C_2^U} \quad (R - C_1) - \overline{\psi} + \overline{p} u\left(C_2^S\right) + (1-\overline{p}) u\left(C_2^U\right) \tag{3.13}$$

$$\text{s.t.} \quad \begin{cases} C_1 + \overline{p} C_2^S + (1-\overline{p}) C_2^U \geq (1-u_1) w_1 + \overline{p}(1-u_2^S) w_2^S + (1-\overline{p})(1-u_2^U) w_2^U \\ u(C_2^S) - u(C_2^U) \geq \dfrac{\psi}{(1-q_1)\Delta p} \end{cases}$$

where $\overline{\psi}$ represents the cost of exerting high effort, $\overline{p}$ and $\underline{p}$ being the respective probability of becoming a skilled worker when exerting and not exerting the training effort. $\Delta p \equiv \overline{p} - \underline{p}$.

Such a model allows studying the extent of intra- and inter-generational employment insurance. This seems to be empirically relevant as evidences for both type of insurance can be found in France for example. Indeed it appears that the employment benefit are, in France, equal to 75% of the last gross wage for the lowest wage bracket and about 57% for the highest one. This is in step with intragenerational insurance between skilled and unskilled workers. As these unemployment benefits are larger for workers above 50 years old, this intragenerational insurance moreover seems to combined with an intergenerational one.

In this setting, our model highlights a tradeoff between training effort and intergenerational insurance. Farsighted agents then prefer to rely on intergenerational insurance rather than on training to deal with the risk of having longer unemployment duration in second period. This application therefore seems to be linked to the actual debate on unemployment insurance (in particular in France) about the tradeoff between unemployment benefits generosity and training subsidization.

Moreover, it appears through the preceding analysis that an increase in the efficiency of training or a decrease in its cost enhances the optimal incentive compatible intragenerational insurance (by decreasing the spread between C_2^S and C_2^U). Finally, our work suggests that an increase in agents' degree of foresight decreases the redistributive pattern of unemployment insurance between skilled and unskilled workers.

3.10 Conclusion

We highlight in this paper the role of prudence and risk aversion on optimal dynamic insurance contracts. To do so we define the notion of foresight as the difference between the index of absolute prudence and twice the index of absolute risk aversion. Adding to usual models an effort of primary prevention, we show that this notion plays a central role in defining the optimal level of prepayment of premia and the optimal incentive compatible classification risk. First, our analysis states that moral hazard always increases classification risk (relative to the complete information benchmark) and increases first period premium (and thus may lead to more prepayment of premia) if agents are farsighted. This reveals the tradeoff between primary prevention and (intergenerational) insurance that arises from future uncertainty.

It moreover appears that the classification risk can be reduced by decreasing the cost of prevention or by increasing the effectiveness of prevention (when agents are farsighted). Therefore, if it aims at making insurance more affordable to high risk agents, the policy maker should seek at reducing the cost of primary prevention and at increasing its efficiency.

Specifying CRRA (Constant Relative Risk Aversion) preferences we moreover show that an increase in agents' degree of foresight decreases the premium offered to low risk agents in second period, if the cost of preventive effort is low enough. Then, the more far-sighted its policyholders, the more stable the mutual insurer when confronted to competing companies that offer short term contracts. To go further in the analysis of comparative foresight, we specify a utility function that exhibits the suitable property of having a linear reciprocal derivative. With such preferences, it appears that an increase in agents' degree of foresight optimally increases first period premium and decreases second period premia. Heterogeneity in behavior toward risk therefore appears as an alternative explanation of the properties of dynamic insurance contract observed by Hendel and Lizzeri [36]. We indeed find that such heterogeneity can explain the different level of prepayment and the fact that contracts with higher levels of prepayment is associated with lower lapsation. Whether this explanation is more relevant that the heterogeneity in income growth put forward by Hendel and Lizzeri [36] remains an open issue.

Is left for future research to analyze the impact of heterogeneous foresight on adverse selection. Preliminary work on this refinement highlights the importance of the relative time preferences. It indeed appears that cross subsidization increases prepayment for the agents with the lower preference for future and is therefore likely to stabilize the dynamic contract. In future work, it would then be worthwhile to analyze the impact of heterogeneous degrees of foresight on this cross-subsidization. Our analysis suggests that agents with a low degree of foresight would subsidy more farsighted agents. In this case, the coexistence of mutual dynamic insurance with spot market (in first period) would be explained by heterogeneous behaviors toward risk, the dynamic form being designed to insure the most farsighted policyholders. However, this effect being coupled with the time preference effect we just discussed, the effect of cross-subsidization is ambiguous. It therefore seams that the study of adverse selection in dynamic contract call further assumptions and probably for the full specify of the utility functions.

It also seems interesting to study the role of foresight on the optimal level of effort. In future work it would indeed be worthwhile to analyze if more farsighted agents exert more primary preventive in line with the work of Jullien et al. [40] on the link between risk aversion and (secondary and tertiary) prevention. Our work also seems to opens perspectives on the role of information. The information on risk type takes an essential role in the writing of dynamic insurance contracts. It therefore seems natural to study the impact of genetic testing and medical checkups in this area. Following Barigozzi and Henriet [4], policyholders would then choose on the one hand to undertake or not the tests and on the other hand to reveal or not the results. The objective of future work would then be to study the impact of these choices on the optimal level of effort and on the optimal dynamic contract. It indeed seems interesting to study the value of information through the trade-off between the resulting increases in effort and classification risk.

Appendix: Simulations with CRRA preferences - The impact of foresight on first period premium

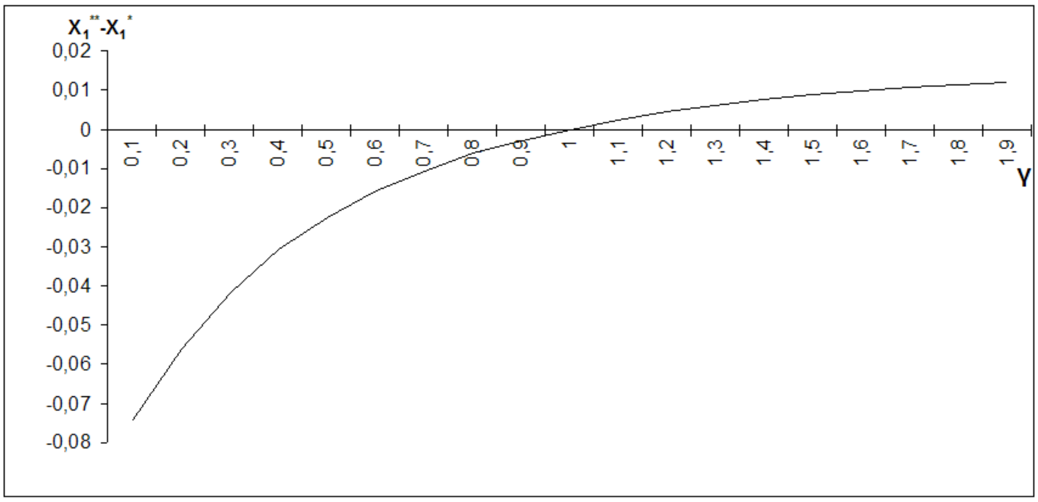


Figure 3.8: $\frac{\psi}{\Delta p} = 1$, $\overline{p} = 0,9$

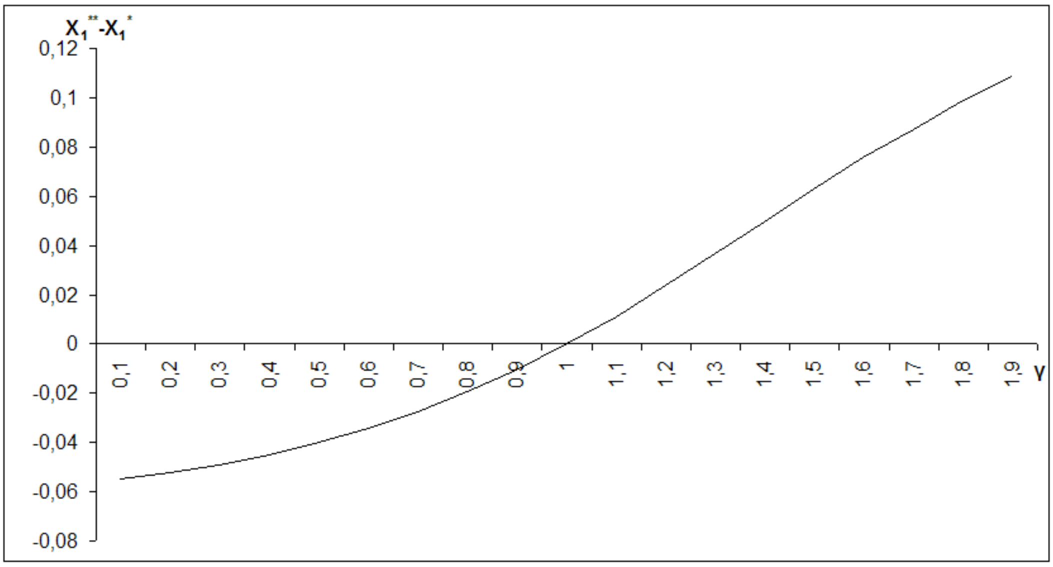


Figure 3.9: $\frac{\psi}{\Delta p} = 1$, $\overline{p} = 0,2$

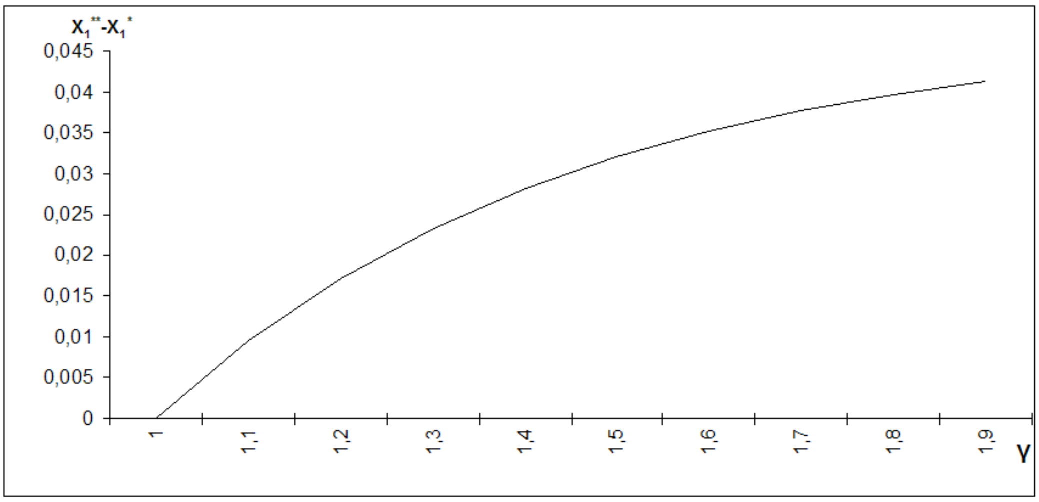


Figure 3.10: $\frac{\psi}{\Delta p} = 10$, $\overline{p} = 0,9$

Conclusion générale

L'assurance mutuelle, plus ancienne solution utilisée pour se couvrir face au risque, continue de dominer certaines lignes d'assurance malgré l'apparition du calcul actuariel et de compagnies d'assurance privées. Le secteur de l'assurance santé dans les pays Européens et la plupart des mécanismes d'assurance dans les pays en voie de développement sont ainsi principalement régis par des accords d'assurance mutuelle. Même dans des secteurs plus classiques, comme l'assurance vie, les organismes d'assurance mutuelle coexistent avec des compagnies privées. Les récentes vagues de mutualisation et de démutualisation aux États-Unis ont par ailleurs démontré que la préférence pour chaque forme d'organisation est instable.

Ces constats nous ont amené à aborder dans cette thèse la question de la stabilité des accords d'assurance mutuelle. Sur la base des principales différences entre les deux formes d'organisation (en termes d'objectifs, d'origine du capital et de définition du risque), cette question à été abordée dans trois contributions grâce auxquelles nous avons pu apporter des éléments de réponse aux trois questions suivantes :

- Sous quelles conditions un accord d'assurance mutuelle peut-il résister à l'entrée sur le marché d'une compagnie d'assurance privée ?
- Quelle forme un accord d'assurance bilatérale doit-il prendre afin de résoudre le problème d'anti-sélection ?
- Comment une mutuelle peut-elle faire face au phénomène d'écrémage des bons risques, en présence d'aléa moral ?

Le premier chapitre de cette thèse a été consacré à l'étude du choix optimal d'une compagnie privée cherchant à assurer une population déjà protégée par un accord d'assurance mutuelle. Ce travail complète la littérature existante en modélisant les choix d'investissement de la compagnie d'assurance. Il intègre ainsi une différence fondamentale entre les deux formes d'organisation : alors que les accords mutuels ne reposent que sur les capitaux de ses assurés, une compagnie d'assurance peut avoir recours à des capitaux extérieurs. Contrairement aux assurés d'une compagnie privée, les membres d'une mutuelle sont donc toujours exposés au risque macroscopique. Une compagnie privée s'engageant sur le versement d'une couverture fixe, le capital qu'elle possède peut cependant s'avérer insuffisant pour tenir cet engagement et la compagnie est exposée à un risque de faillite. Ces phénomènes induisent une relation explicite entre le stock de capital d'une compagnie et les contrats d'assurance qu'elle offre.

En prenant en compte cette interdépendance, les choix optimaux d'une compagnie d'assurance devant composer avec un accord mutuel préexistant ont été définis dans le premier chapitre. Dans le cas d'agents homogènes, un tel exercice permet d'exposer les conditions sous lesquelles une compagnie privée ne peut pénétrer un marché assuré par des accords mutuels. Il apparait ainsi que la possibilité pour une compagnie d'assurance de concurrencer un accord mutuel est d'autant plus faible que la taille de la population assurée et le coût du capital sont importants, et que le risque et l'aversion au risque des agents sont faibles.

Le modèle présenté dans le chapitre 1 fournit ainsi une explication à la présence exclusive des mutuelles dans certaines régions ou lignes d'assurance. L'hypothèse d'agents homogènes implique cependant l'omission d'une autre caractéristique importante des marchés de l'assurance, à savoir la coexistence des deux formes d'organisation.

Le deuxième chapitre a ainsi concerné l'étude du relâchement de cette hypothèse et plus particulièrement de ses conséquences sur les contrats d'assurance mutuelle. Il a s'agit d'étudier l'accord que deux agents exposés de manière différente au même risque doivent mettre en place afin que chacun d'entre eux soit incité à révéler l'information qu'il possède sur sa probabilité de sinistre.

Dans le cas d'agents exposés de manière similaire au risque (comme dans le chapitre 1), grâce au principe de diversification, il est optimal de diversifier totalement le risque c'est-à-dire de partager également les pertes. En considérant des agents hétérogènes face au risque, il a cependant été démontré que cette règle de partage n'est pas toujours compatible avec la participation des individus les moins exposés au risque. Si l'hétérogénéité est trop importante ou si les agents sont trop peu averses au risque, les individus les moins exposés au risque préfèrent ne pas s'assurer plutôt que de partager égalitairement les pertes. Comme précisé dans le chapitre 2, un accord mutuel garantissant la participation des "bas risque" peut tout de même être trouvé. Si l'information sur le risque est parfaite, cet accord satisfait le principe de mutualisation, l'allocation ex-post des richesses ne dépendant que de la richesse agrégée.

Ce n'est toutefois plus le cas quand on suppose que l'information sur l'exposition au risque est privée. Dans ce cas, l'accord mutuel doit être considéré comme un mécanisme incitant les agents à révéler leur information privée. Il apparaît alors nécessaire, afin que les "haut risque" ne soient pas incités à se faire passer pour des "bas risque", de rendre l'allocation ex-post des richesses dépendante des richesses individuelles ex-ante. L'asymétrie d'information conduit alors à la récusation du principe de mutualisation. Notre deuxième contribution a ainsi consisté à proposer une explication alternative à la faillite du principe de mutualisation observée par la littérature empirique, et jusqu'ici principalement expliquée par des limites dans l'engagement des agents.

Le chapitre 2 a par ailleurs contribué à préciser l'avantage comparatif que possèdent les accords d'assurance mutuelle en termes d'information. Il semble en effet d'après cette étude que les accords mutuels soient mieux à même de gérer les problèmes d'information que les compagnies privées. Si une asymétrie d'information conduit toujours à une perte d'efficience dans le cas d'une compagnie d'assurance (cf. Rothschild et Stiglitz [62] et Stiglitz [65]), il a été montré dans ce chapitre que ce n'était pas le cas pour les accords d'assurance mutuelle. Ce résultat est cohérent avec les observations générales qui veulent qu'une compagnie d'assurance puisse toujours offrir une meilleure couverture qu'un accord mutuel dans un cadre informationnel parfait, mais qu'elles ne peuvent assurer un risque sans information, contrairement aux mutuelles.

La troisième contribution, présentée dans le chapitre 3, a prolongé l'analyse des conséquences de l'hétérogénéité face au risque à travers l'étude du risque de classification, qui correspond, dans le cas de l'assurance de long terme, au risque d'être classifier "haut risque" par son assureur et donc de payer une prime (trop) élevée. Particulièrement important dans le domaine de la santé, ce risque semble pouvoir être réduit grâce à une mutualisation inter- et inter-générationnelle du risque. Un tel accord apparaît toutefois particulièrement instable, principalement à cause des phénomènes d'écrémage des "bon risque", et peut mener à des effets pervers d'aléa moral si des efforts de prévention primaire sont modélisés.

L'étude de ces phénomènes dans le chapitre 3 a mis en évidence le rôle des préférences des agents (en termes de prudence et d'aversion au risque) dans la détermination des efforts de prévention primaire et d'assurance intergénérationnelle. Cette analyse a en effet permis de différencier deux types de comportements face au phénomène d'aléa moral. Les agents dits "prévoyants" (dont le coefficient absolu de prudence est supérieur à deux fois le coefficient absolu d'aversion au risque) préfèrent utiliser l'assurance intergénérationnelle plutôt que la prévention pour faire face au risque futur. En présence d'aléa moral, pour inciter ce type d'agents à l'effort, il sera donc nécessaire d'augmenter le risque de classification. Au contraire, dans le cas d'agents dits "non prévoyants" privilégiant l'effort aux transferts intergénérationnels, la prise en compte de l'aléa moral réduit à la fois le risque de classification et l'assurance entre générations.

Le chapitre 3 apporte par ailleurs des enseignements quant à la stabilité des accords de long terme. Il apparait ainsi que pour certaines classes de fonctions d'utilité (notamment celles spécifiant un coefficient d'aversion relatif au risque constant) un accord d'assurance de long terme est d'autant plus stable par rapport au phénomène d'écrémage qu'il concerne des agents prévoyants.

Les trois contributions présentées dans cette thèse font référence à des travaux assez indépendants mais dont les résultats, combinés, complètent de manière cohérente la littérature relative à la stabilité des accords d'assurance mutuelle.

Le travail mené dans la présente thèse permet ainsi d'appréhender les paramètres favorisant la stabilité des accords mutuels dans le cas d'agents homogènes et d'analyser les conséquences de l'hétérogénéité sur l'exposition au risque des agents. Il met notamment en évidence le relatif avantage des organisations mutuelles quant à la prise en compte des problèmes d'asymétrie d'information.

En complément des pistes de recherche présentées en conclusion de chaque chapitre, plusieurs projets de recherche sont envisagés à la suite de cette thèse et concernent principalement le rôle de l'information.

La majorité des travaux relatifs à l'information en assurance considère le cas d'agents ayant une connaissance parfaite de leur probabilité de sinistre. Il semble cependant qu'en réalité, les assurés ne possèdent qu'une information imparfaite sur celle-ci et ne la découvre qu'à force d'observations, tout comme leur assureur (les cas de l'assurance santé et de l'assurance automobile sont particulièrement représentatifs de ces phénomènes). Il apparaît ainsi intéressant d'étudier les processus "d'apprentissages de risque" et leur impact sur les contrats d'assurance.

Dans le cadre d'accords mutuels, cet apprentissage est couplé à un apprentissage du risque des autres membres, ou du moins de risque moyen dans l'organisation. C'est alors en comparant l'information récoltée sur son propre niveau de risque à celle du reste de son organisation, que l'agent peut évaluer la profitabilité de l'appartenance à une mutuelle. Il s'agirait ainsi de relier les travaux de Jackson, Kalai et Smordoninsky [39] sur l'inférence bayesienne aux théories de la décision en assurance.

La difficulté vient ici du "double apprentissage", puisqu'à chaque période l'assuré réactualise sa croyance sur son propre risque mais aussi sur le risque moyen de son organisation. Ces croyances sont par ailleurs d'autant plus précises que le nombre d'observations est grand. Il convient alors à l'assuré de comparer la satisfaction qu'il obtiendra seul, à celle qu'il retire en appartenant à la mutuelle, compte tenu de cette précision. L'étude de ce type de phénomènes permettrait ainsi de compléter l'analyse de la stabilité des accords mutuels. Il apparaît en effet intéressant d'étudier le nombre d'observations nécessaire à la décision, en fonction de la marge d'erreur que s'accordent les assurés et des différents degrés d'exposition au risque des individus considérés. Les décisions passées des agents les moins exposés au risque influençant les décisions présentes des agents restant dans l'organisation, ces paramètres joueront un rôle majeur dans la stabilité de l'accord. Des simulations préliminaires réalisées avec une distribution uniforme des expositions au risque et des révisions bayesiennes des croyances suggèrent qu'un partage égalitaire est alors stable (dans le sens où une grande partie des agents reste dans l'accord après un grand nombre d'observations) quand les individus sont fortement averses au risque. Cependant, dans le cas d'agents faiblement averses au risque, une organisation mutuelle mettant en place un partage égalitaire expérimente un phénomène d'anti-sélection très rapide.

Dans un second projet, il semble intéressant d'utiliser les avantages comparatifs des deux formes d'organisation afin d'étudier leur possible complémentarité. Il est apparu au long de cette thèse que l'un des principaux désavantages des accords mutuels (par rapport aux compagnies d'assurance) réside dans leur incapacité à assurer le risque macroscopique/agrégé. Le chapitre 2 a toutefois confirmé que cet inconvénient peut-être balancé par l'avantage que les accords mutuels possèdent dans la prise en compte des problèmes d'information.

Il apparaît donc possible d'augmenter l'efficacité des accords mutuels tout en profitant de leur capacité à gérer l'asymétrie informationnelle, en les combinant avec une organisation privée qui fournirait l'assurance macroscopique manquante. La compagnie privée assurerait ainsi le revenu agrégé de l'organisation mutuelle (par exemple la coopérative d'un village), qui aurait alors en charge de redistribuer ce revenu en fonction des réalisations des risques microscopiques/individuels de ses membres. En prolongement des travaux de la présente thèse, il semble ainsi intéressant d'étudier la forme optimale de tels accords en présence d'aléa-moral et/ou d'anti-sélection.

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